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ABSTRACT

Seven papers presenting recent findings on mathematics and science education are included in this report. Most of the papers deal with improving the mathematics/science instruction for minorities and women, and were presented at a series of seminars for the improvement of teaching math and science. Paper titles (and authors) are: (1) "Neurological Potential and Mathematical/Scientific Abilities" (Roger S. Gurney); (2) "Minorities and Mathematics: Why So Few?" (Westina Matthews); (3) "Blacks and Women in Mathematics" (Patricia C. Kenschaft); (4) "Administrative and Curricular Issues in Mathematics Education of Minorities and Women" (Joan E. Coleman); (5) "The Role of State Education Officials in Expanding Educational Opportunities in Mathematics and Science" (Julia Lara); (6) "Addressing the Crisis in Precollegiate Mathematics and Science Education" (Bernard R. Gifford); (7) "Toward a Mathematics Curriculum in the Form of the Core and Option-Modules" (Hiroshi Fujita, Fumiyuki Terada, and Shigeru Shimada). An introduction and brief profiles of contributors are included. (KH)

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FOREWORD

Jane Matthews

Today the media, scientific associations, business leaders, and governmental commissions at all levels are expressing grave concern for improving mathematics and science education. The focus varies. It may be on increasing the number of teachers through incentives and retraining, on tightening standards for students and teachers, or on increasing the numbers of women and minorities. However, there seems to be agreement on three points. (1) The need for improving mathematics/science education is urgent. (2) Massive federally-funded programs which aided educational and career opportunities through affirmative action are no longer available. (3) The pitfalls of introducing programs too hastily must be avoided. Such haste prevented the post-Sputnik programs for the "new" mathematics and science from having broad, long-term, positive effects.

I see a need for increased recognition of the idea that, to be truly effective, *all* groups must work together in developing, supporting and implementing reforms agreed upon in light of what we should have learned from the past. But how? We must find ways to increase communication among groups with different emphases. Local projects such as the College Discovery and Development Program (CDDP) Urban Project—working with targeted high schools under the auspices of The City University of New York and the New York City Board of Education—provide important experimental bases whose methods and results must be broadly disseminated. Large-scale programs can then adapt the best of the pilot projects and join in channeling information for continued strength and growth.

I believe this publication is uniquely valuable because it brings together results of research efforts that otherwise might have rather limited and separate audiences. Hopefully, it will help to stimulate further positive interactions. It is certainly time we faced the inadequacy of mathematics and science education for an age in which every one of us is increasingly affected by science and technology, both in our daily lives and in our concerns as citizens in a democracy. In the past we could survive as a nation and a world with a relatively small scientific community, mostly male and white, trained for the most part within a small number of specialized schools. (Those specialized schools, incidentally, included some in the New York City public school and the City University systems where the number of graduates going on to outstanding careers in science has long been among the highest in the nation. Some of these graduates also helped to achieve a slight improvement in the ethnic balance in the science community.) We must make the appropriate changes in our *total* educational system, from its beginnings in the home and local community through high school and college in order to fill the gap that now requires us to import so large a portion of our graduate student population in the sciences.

I believe we must first examine research results, such as those presented in this publication, which are not widely known nor readily available, if we are to consider adequately the competing—if not actually conflicting—opinions being presented from various sources.

Dr. Gurney's paper gives us specific examples of research that appears to have important implications for teaching. It makes available to us research information on the structure of the brain and its relationship to achievement in mathematics and science. We learn of accomplishments that seem physically impossible and are encouraged not to set our sights too low for *any* student. Dr. Gurney points out the importance of educational sequencing and methodology that takes into account the spurts of brain development so that the potential at each stage of brain growth is not wasted or called upon prematurely.

As the paper by Drs. Fujita, Terada, and Shimada clearly indicates, the national expectations for the general population in the United States and in Japan are currently very far apart, although perhaps moving closer together both in terms of a balance between setting standards

and encouraging local flexibility and in deciding which mathematics topics should be included, for whom, and at what level. Thus, for example, the probability and statistics that used to be reserved for graduate school are successfully and profitably being introduced even at the elementary school, while the presentation of some traditional high school algebra and trigonometry topics might perhaps be delayed until students study their applications. With these kinds of curriculum changes students can be better motivated to move beyond an essentially rote level of knowledge to the level of relating that knowledge to effective problem solving.

Dr. Jewel Plummer Cobb, biologist and educator, used the analogy of the laboratory filter in discussing what has served at various stages to limit the number of women entering careers in mathematics and science. She delineated six "filters" in the educational experience of a young woman scientist: socialization in the preschool years, elementary school, junior high, high school, college, and graduate school. She regarded the high school years as the most critical period and emphasized the central role that academic advisers play in helping young women maintain their interest in science.

Dr. Gifford raises a similar issue in his paper and calls for the involvement of the scientific community in keeping teachers and curricula up to date, and in better informing advisers and counselors at high school and college levels.

Much valuable research on women in mathematics has been conducted by women themselves. Many successful programs for women in science have been directed by women. Consequently, I applaud Dr. Westina Matthews' call for more members of minority groups to be involved in designing and carrying out research studies on minority involvement in mathematics.

Dr. Matthews' article brings out the important, but perhaps little known, point that minority students tend to like mathematics and have little of the "math anxiety" that is often cited as a cause of underrepresentation of women in mathematics and science. For minority students who opt out of further study, the cause is frequently lack of awareness of the relevance of classroom mathematics to their career aims. Perhaps we need a publication like *How to encourage girls in math and science* (Skolnick, Langbort, and Day 1982) that would give parents and elementary school teachers of minority students materials relevant to their inner city or rural environments, and thus help prevent attendance and discipline problems from limiting the school's effectiveness.

As Sue Berryman notes in *Who will do science*, we must enlarge the initial mathematical/scientific talent pool and find ways to limit migration of students out of that pool; this involves not only determining why students leave but, as Dr. Matthews suggests, conducting research on those who stay and succeed.

Dr. Kenschaft's paper provides examples of successful black mathematicians from diverse backgrounds, examples which show the importance of supportive networks, both of family and friends, and of role models. I believe it is important that Dr. Kenschaft emphasizes the fact of struggle. If professionals—through such models as the Urban Project Summer Institutes and Precollege Institutes at high schools and colleges—share with groups of students, both formally and informally, not only their own struggles, discouragements, and failures, but also their satisfaction with the rewards for persistence, then students may be better able to resist pressures to "migrate out of the pool" and even build up positive pressures to enter or remain in that pool.

Dr. Kenschaft's definition of mathematics in terms of patterns and her reference to the joy of seeing the many applications of one pattern are particularly appealing. So often the question "What is mathematics?" is not even asked, much less answered suitably, before suggestions are made about how to teach it, whatever "it" may be.

Ms. Coleman gives a very comprehensive report on the current problems and touches on nearly every factor that must be considered. She rightly points out that the curriculum is not keeping up with new technology and that the "back to basics" emphasis on competency testing risks minimizing learning and lowering expectations, as it focuses wrongly on computational rather than problem solving needs.

While giving a summary of what is happening at the state level across the nation, Ms. Lara's paper cautions against inappropriate use of merit pay and competency testing for teachers, and cites the importance of role models for minority students.

I heartily endorse Ms. Lara's caution about increased dependence on specialized high schools. These schools are not doing anything to *increase* the pool of well-prepared students, but may be increasing the early tracking which is already a great part of the problem for women and minorities. If specialized and magnet schools succeed in increasing test scores and in lowering absenteeism, we should learn how to spread these effects to all schools rather than further polarizing educational opportunities and making problems worse at non-specialized schools.

As Ms. Lara's remarks imply, important dimensions of the practice of early tracking have yet to be explored. For some educators, such as the late Ed Begle, its benefits for the most able students are beyond dispute. (Dr. Begle was director of the School Mathematics Study Group (SMSG) from 1958 to 1972.) It only remains for further research to determine *which* students should be accelerated as opposed to being offered enrichment programs (Begle 1979, 106). However, as Drs. Fujita, Terada, and Shimada report, for other educators (in their case Japanese) the advisability of early tracking is less clear. They report resistance to the practice based on their society's unwillingness to group students according to aptitude and on their recognition of the fact that students who are "slow starters" can catch up with and surpass their peers in the first year of high school. The subject of early tracking, then, is just one example of a pedagogical topic that provides a ready basis for fruitful discussion and exchange on the international level.

It is fascinating to consider which topics and approaches these Japanese scholars consider relevant to the core versus the option modules, remedial or enrichment. Particularly during the current period when curriculum changes are being considered, I believe it is important to remind ourselves of the limitations of available standardized tests as an aid to improving mathematics education (Begle 1979, 11).

Dr. Gifford reminds us that we must deal with the isolation from new disciplinary developments that faces beginning science and mathematics teachers, and that, unless this problem is dealt with, we shall continue to lose some of our best teachers to higher-paying careers. He also alerts us to the demographic fact of increasing proportions of immigrant and minority students in our school age populations. In the past such students took fewer mathematics and science courses. However, I would note that, at least in lower Manhattan, these proportions have long been high and sometimes language barriers have led students *into* mathematics and science where language was not as great a barrier to learning as in other disciplines.

Dr. Gifford's idea of using "advanced, pedagogically qualified graduate students in mathematics, science and technology to take on part-time teaching assignments, especially at the junior high and secondary school level" is indeed radical. In our Problem Solving course at Hunter College, we use graduate and undergraduate science students as instructors and tutors. They provide positive role models, career motivation, and a determination to require high standards of the students. Some difficulties may occur when both instructor or tutor and student have learned English as a second language. Yet here, too, the persistence of an instructor or tutor, despite language difficulties, may in itself serve as a constructive role model. There is, of course, a need for careful training, particularly when graduate students are used at precollege levels, so that they may be aware of their students' stages of intellectual and social development and their needs at each stage, as suggested by Dr. Gurney and others.

The improvement of education in mathematics and science will require hard work by many. We encourage all who read this publication to react to it by becoming active participants in the cooperative effort that is the hope for today and the future.

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We also thank the Urban Project staff—college and high school teachers, counselors, and administrators—for their continuing support of the faculty development activities that fostered this publication.

And we thank the students; their enthusiasm and perseverence inspire us all.

INTRODUCTION

Joseph R. Harris

The College Discovery and Development Program (CDDP) was originally established by the Board of Trustees of The City University of New York in 1964 to foster the expansion of educational opportunities for disadvantaged youth by providing college-level and college preparatory classes, individual and group tutoring, and group career and life planning instruction to approximately 1,500 high school students. The program operates in five high school centers (one in each borough of New York City) and receives assistance from the City University through its faculty and students in four critical areas: curriculum development, tutoring, counseling, and program research and evaluation. These services are rendered through a consortium between the New York City Board of Education's Office of Funded Programs (in the High School Division) and the Office of Special Programs of The City University of New York.

In 1981 the Consortium requested support from the Ford Foundation to plan and initiate an experimental secondary-postsecondary articulation program—the Urban Project: Minority Students' Advancement in Sciences and Technologies—providing some 100 students with an opportunity to develop their potential in mathematics and science. Since that time, over 1,000 CDDP and other urban high school students have benefited through their participation in one or more of the seven specific educational interventions jointly developed by the Board of Education and City University staff. The seven interventions included: Bridge Projects, Science and Technology Field Trips and Seminars, the Summer Mathematics and Science Institute, Science Clubs, Tutoring, Parent Workshops, and special projects. All of these interventions were designed to accomplish two Urban Project objectives: (1) to identify a group of minority high school students with underdeveloped potential for mathematics and science careers; and (2) to design an urban educational model through which the students could advance toward careers in sciences and technologies. The first group of Urban Project student participants entered the University in Fall 1984 and a report on their freshman year experience is forthcoming.

The science and technology field trips and seminars, one of the original educational interventions, were intended primarily to serve students. However, after noting the intense interest and enthusiasm of high school and college faculty who accompanied students on a field trip to the Educational Testing Service facilities in the Spring of 1982, the Urban Project staff decided to develop a series of seminars devoted to faculty and to others concerned with improvement of mathematics and science education. Speakers met with faculty and staff to discuss the crucial roles teachers and administrators play in improving mathematics and science education for students. In addition, presentors attempted to enlarge seminar participants' understanding of the relationships between themselves and events in the areas of scientific, technological, and societal change. As a result of benefiting from this positive interaction, seminar participants suggested two ways of sharing our experiences: to publish the seminar presentations and to continue the dialogue among teachers and school administrators through a Faculty Development Program.

Since the decision was made to publish some of the original presentations, three other excellent papers were brought to our attention. Through the kind cooperation of several outstanding professional colleagues and their institutions, we are able to include their significant contributions. One (Lara) addresses the steps being taken by state governmental authorities to improve mathematics and science education. Another (Gifford) directs our attention to the need to involve makers of public policy in our dialogue, especially lawmakers at the national level. Finally, through the efforts of our Japanese colleagues (Fujita, Terada and Shimada), we learn that our local efforts to address critical issues in mathemat-

ics and science education also have an international dimension. We are indebted to each of these contributors for sharing their ideas and practical advice with us.

You, the readers, are the beneficiaries of these thoughtful essays and hopefully they will stimulate you to join us in our efforts to promote locally, nationally, and internationally the improvement of mathematics and science education. In the final analysis, it is our intention that these papers will stimulate thought, debate, and positive action among student and faculty readers for the benefit of all learners.

More than most efforts, the seminar series and this publication in particular reflect the hard work and dedication of Urban Project staff. I am especially grateful to Ms. Barbara Gombach for her coordination of this project.

NEUROLOGICAL POTENTIAL AND MATHEMATICAL/SCIENTIFIC ABILITIES

Roger S. Gurney

Introduction

In this paper, I want to address a number of topics concerning our current understanding about brain structure, brain growth, and brain functioning. These all focus upon academic achievements, notably in mathematics and science. Most of the studies referred to are comparatively recent and not widely known. The implications drawn from them are often frankly speculative and yet, nonetheless, are of such potential significance that we ignore them to our cost and especially at a cost to the children who might ultimately be the beneficiaries of the programmes and new curricula that would emerge from a reassessment of the current position. This is to say no more than that we appear to be at the start of an era when the burgeoning information from new techniques and new technologies, many largely medical in orientation, will provide challenging ideas and questions for those educators prepared to consider the educational implications and consequences.

Brain Structure and Functions

The adult human brain has vast numbers of cell units, neurons, which are collectively responsible for the psychological processes involved in learning, perception, memorization, and problem solving, the functions which underlie scientific and mathematical abilities. There are about ten billion of these active, interconnected processing units and they are surrounded by ten times this number of support cells. The second type, the glial cell, is thought likely by some authorities to make its own contribution to the overall psychological capabilities of the brain. One suggestion has it that the glial cells may act like memory units in microprocessors (Russell 1979).

This huge array of brain cells gives a mass of approximately 3 lbs. in weight, most of it collected into an enormously enlarged fore-brain consisting of left and right hemispheres. These hemispheres are divided into four lobes which specialize in the organization of particular types of function:

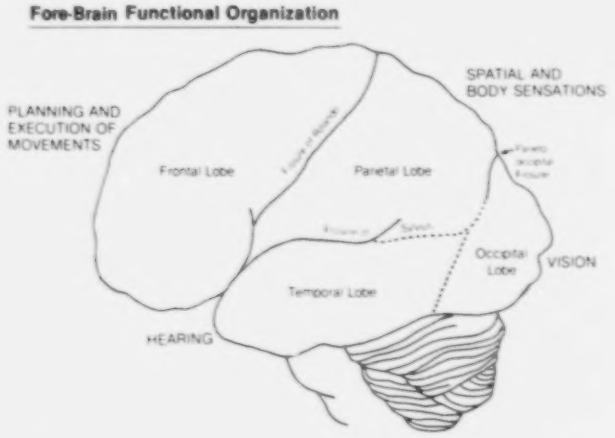

Figure 1.

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It is thought that most of the cells concerned with the organization of these general functions are packed into a dense outer layer some 5 mm. in thickness, the cerebral cortex. In addition, it has been known since the latter half of the nineteenth century, when the French doctor Paul Broca discovered that a portion of the left frontal lobe was responsible for the organization of speech, that there are key areas within each lobe which have a specialist role. Broca's discovery of a speech area in the left hemisphere was the first scientific evidence for what have become two key concepts in present-day psychoneurological knowledge. These are that: (1) Specific areas of the cortex mediate specific psychological functions and (2) the two hemispheres make different contributions to the organization of psychological processes.

Both concepts are part of our present understanding about how different parts of the brain are involved with the proper development of mathematical abilities. We know, for instance, that silent, digital counting maps to the frontal cortex of the left hemisphere (Roland, Larsen, Lassen, and Skinhøj 1980). This is in line with the generally held view (Springer and Deutsch 1981) that the left brain prefers a sequential, logical, and analytical mode of operation, which underlies mathematical calculation or algebraic problems. Similarly, geometric tasks and the interpretation of graphs, which have strong visual components, are thought to be organized through the right hemisphere.

But the well-established emphasis on separate left- and right-brain processes has given way to a more recent conceptualization of complementary action between the two hemispheres. As has been pointed out (Kuyk 1982), numerical and spatial concepts require the simultaneous and complementary activity in both hemispheres. Natural number has both ordinal and cardinal elements and these are mediated through the separate halves of the brain. The ordinal aspect involves sequential processing in the counting of digits, for instance, whereas the cardinal aspect is concerned with holistic processing in creating the image of a set. Both processes are involved in the pupil's overall conception of a natural number.

Technology, Brain Structure, and Mathematical Potential

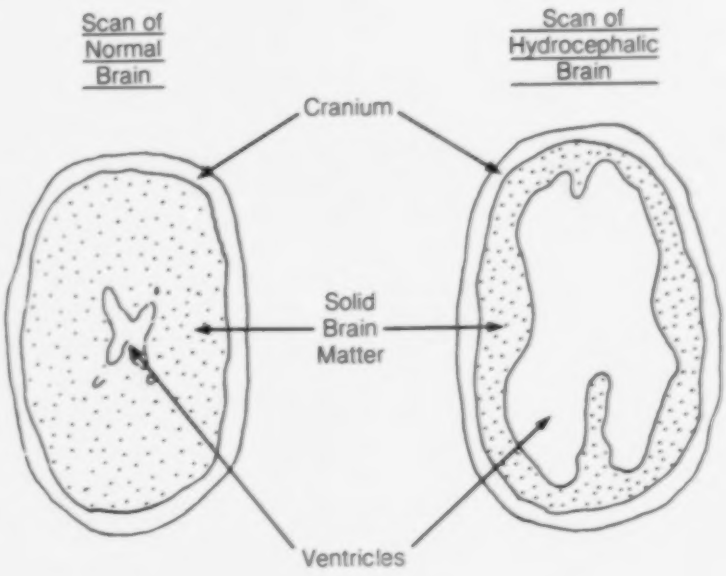
New techniques of studying brain activity are largely responsible for the view that various sites in the brain are simultaneously active when a person is engaged in a task, such as reading or mathematical problem solving. Prior to this the structural information about children with discalculia or dyslexia has had to be gleaned from gross behavioural observation and post-mortem observation. Similar limitations have obtained for recording brain activity. EEG (electroencephalograph) methods have been available in which paired electrodes are fixed to the scalp thus measuring minute electrical fluctuations from millions of cells in the underlying region of the brain. While neither approach is entirely satisfactory, both have been used in studies which indicate unusual patterns of growth and activity in children with learning problems. In fact, this kind of explanation has been put forward as a general factor to account for the poor showing of many girls in mathematics at the secondary level (Badger 1981). And, while this particular explanation is not likely to be the whole story, it does seem to be possible that certain patterns of brain activity are indicative of superior or inferior academic processing. For instance, older children with specific learning difficulties have weakly lateralized language functions (Newton and Thomson n.d.). In these pupils, language processes seem to be more equally distributed in the two hemispheres than is to be found in their normally functioning peers.

Now the living brain can be seen in action with some precision and effectiveness. A new technology which combines the established physiological technique of using radioactive isotope traces with gamma-ray cameras and the use of a small digital computer has been developed. This, the PET scan (positron-emission tomography), gives information which can be displayed in graphical form on a colour-television monitor. PET scans confirm the selective activation of sites in the two hemispheres as a result of increased blood flow dur-

ing mathematical tasks. There would appear to be a number of sites associated with each mathematical function in each hemisphere (Kuyk 1982).

A combination of techniques was reportedly used in a follow-up study on children born with the hydrocephalic condition in Sheffield, England (I.T.V. 1982). In it, a number of people were examined using the new methods and the study also reviewed their educational abilities and attainments. These people, now in their late teens or early twenties, had all (with one exception) been operated upon in early childhood to alleviate the hydrocephalic condition. Scans which indicate soft tissue, such as the brain, now make it possible to see structures directly. In most cases, a uniform pattern emerged; the brain size was of normal proportions but with its central cavities very much enlarged. As a consequence, the neural matter was disproportionately small and, in one case, was estimated to be only 5 percent, by weight of a normal adult's.

Figure 2



Of particular interest was the fact that these individuals were normally-functioning pupils within their secondary schools, despite the apparent lack of brain matter. In fact, some very superior academic attainments were recorded against their names. A young man with one of the lowest estimated brain weights had been good enough to receive a first-class honours degree in mathematics.

This, then, represents a very intriguing situation. How is it possible to demonstrate such formidable learning abilities with so much of the normal brain material missing? PET scans were carried out to try to resolve the problem. These demonstrated very high levels of neural activity in the remaining material, with occasional relocations of functional sites. At least two possibilities exist: (1) It could be that we can function quite effectively on a small volume of brain cells, provided they are not damaged. Such an interpretation has tremendous implications for the potentiality of attainment in all individuals, including the more handicapped; (2) it could be that the brain operates a policy of tissue sparing under adverse conditions. This is a well-known biological phenomenon in which the most important tissues and organs are selectively spared, usually under conditions of starvation. In this case, it is conjectured that the remaining brain mass consists predominantly of neurons, thus accounting for the high level of processing activity. Under duress, occasioned by the enlargement of the central cavities, the support cells are thought to be lost. Whichever is the case, the study provides ample opportunity for adopting a positive stance toward the learning potentiality for all individuals.

Brain Development and Longitudinal Studies of Mathematical Abilities

One of the most interesting neurological conceptions to arise in the last decade, and certainly one with a very direct application to curricular matters, is the notion of brain spurts and plateaus (Epstein 1978, 1981). From birth to the start of adulthood at eighteen years of age, five special spurt periods have been identified through an examination of the data from various longitudinal studies. These spurts occur at approximately 3-9 months, 2-4 years, 6-8 years, 10-12 years, and 14-16 years. During the latter four periods, any individual is likely to undergo a growth spurt during one six-month phase. That is, over the two year period, most of the children (up to 90 percent) undergo a rapid growth of brain weight at some point. In the intervening plateaus, little or no additional brain growth occurs.

Epstein has shown very convincingly that there is a corresponding pattern of intellectual progress which accompanies the pattern of brain growth. In particular, the structuralist account of intellectual and cognitive development which Piaget proposed is seen to have very close links with the phasing of the growth and plateau periods. The start of each growth period coincides with the start of the major phases delineated by Piaget. For instance, the 6-8 growth period corresponds to the change from pre-operational to concrete thinking at the start of formal schooling. It is a point when the rapid period of neural growth, involving the establishment of new functional systems of neural elements, may be converted into significant changes in learning. And, since Piaget identifies many of the intellectual structures or schemas developed at such a time as essentially mathematical, we have here a direct link between neurological and mathematical ability.

But the link is only a potential relationship of interrelated growth, since there is evidence, particularly from the later brain growth period, that many pupils underfunction by not achieving the corresponding intellectual stage. For instance, in looking at the mathematical/scientific abilities of secondary school pupils, one study found that only about 30 percent of those pupils had reached the formal operational stage of logical thinking by sixteen years of age (Shayer et al. 1976). Since there appears to be no convincing reason why the majority of pupils should not reach a fully logical stage of ability, the shortfall points to untapped potential in the school population and to a probable failure in teaching methodology.

The pattern of significant underachievement at the secondary level is one which appears wherever such studies are conducted. It appears to be quite widespread in that a number of countries, such as the U.S.A., England, and Australia, all report similar findings. To such findings must be added as well the extra burden of social and cultural disadvantage. In the field of mathematics, it appears to be a handicap to be female, since girls' abilities tend to fall away when they enter secondary schools. And, although neurological causes have been put forward as the reason for the girls' difficulties, the overriding problems seem to be ones of adequate opportunity and motivation (Badger 1981).

These secondary level problems are made more acute by the revelation that the last growth spurt in mid-adolescence is, for some pupils, the opportunity to move to another level of intellectual ability (Arlin 1975). It represents a significant step beyond formal reasoning and is only available to that third of the school population which has already attained the level of full logical ability. The intellectual abilities demonstrated at this new stage of intellect are ones associated with the posing of original social and scientific questions, of discovering important directions of inquiry to pursue, of demonstrating the potential for creative problem solving.

Conclusion

The new technologies will undoubtedly provide a wealth of new detail on neurological structure and functioning in the near future. Even now we are at a point when the pattern of existing studies, although at times with somewhat tenuous and speculative application to education, is posing fundamental questions about the full potentiality for learning in our pupils. All educators need to be sure that they learn to appreciate the significance of these discoveries and that, at the very least, their curricula and educational strategies are broadly in tune with what is known about the brain's growth and potential.

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MINORITIES AND MATHEMATICS: WHY SO FEW?

Westina Matthews

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Introduction

Based on the results of several national studies, it has been established that blacks and Hispanics are consistently scoring below the national norm at each level assessed, and that they are continuing not to enroll in mathematics courses at the more advanced levels (NAEP 1980; Peng, Fetters, and Kolstad 1981). Between 1972 and 1978 modest improvement was reported for nine-year-old black and Hispanic students; this favorable trend was reversed by seventeen-year-old black students. Although seventeen-year-old Hispanic students did improve their performance slightly, they still scored appreciably below the national norm. Moreover, black and Hispanic high school students take fewer mathematics classes, averaging less than two years of mathematics while attending high school and often these courses are at the introductory level. In contrast, white students average more than three years, often at the more advanced levels (Anick, Carpenter, and Smith 1981).

Although the 1980s hold promises of increased science- and mathematics-related jobs (e.g., computer science, engineering, business), I wonder whether or not the young people will be prepared to take advantage of these opportunities. This question becomes particularly pressing when we consider the following statistics. Nearly 10 percent of all seventeen-year-olds and 40 percent of all black seventeen-year-olds are functionally illiterate (U.S. Task Force 1980). For a youth to be identified as functionally illiterate, he or she has to be unable to follow simple written instructions, unable to read a bill, unable to write a check, and unable to read the amount of payment on a traffic ticket. In very simple but tragic terms, it means that the opportunities for a functionally illiterate black youth will be severely limited.

The learning of mathematics that we are discussing here ranges from the very basic to the very abstract. We are discussing: (1) mathematics needed for everyday survival (e.g., paying bills, counting money), (2) mathematics needed for nonscientific positions (e.g., bank teller, police officer, teacher), and (3) mathematics needed for a science- or mathematics-related job.

Why do minority students seem to be unable to master basic mathematics skills? Why do minority students continually score below the national norms, earn low grades in mathematics classes, and decide not to enroll in higher level mathematics classes? Why have we been unable to find the answers to these questions? Let us examine the past research on blacks in mathematics so that we might begin to understand our difficulty in providing appropriate solutions to the problem.

Past Research

I have just completed a critical review of the research conducted on "minorities and mathematics" since 1975. The year, 1975, appears to be a watershed for the beginning of

research on "minorities and mathematics." This may be because much of the research on "women and mathematics" was well underway, and questions were beginning to be raised about the participation of minority women in mathematics. Most of the studies in the review are inaccessible to the general public. Reports of these studies are usually either unpublished papers (e.g., presented at conferences) or are final reports to funding agencies. In over 75 percent of these studies, blacks were studied as the subgroup, and in 50 percent of the studies, Hispanics were studied. Nearly two-thirds of the studies have been written since 1980.

Based on a critical review of the twenty-four studies, there were a number of variables that have been identified as having an influence on either the student's participation or the student's performance in mathematics. There were three clusters of variables that have been considered: home, student, and school. Home variables included ascribed characteristics (e.g., race, sex, age), parental status and attitudes, environment, and language. Within the student cluster, variables studied were either cognitive (past and present academic performance or enrollment), affective (achievement orientation, aspirations and expectations, stereotyping, etc.), or language (cognitive learning style, language proficiency). The school variables were climate, organization, resources, racial composition, and school personnel.

Home influence. There is considerable evidence available that *race* does have an effect on performance and enrollment (Burton and Jones 1982; Anick, Carpenter, and Smith 1981). As was mentioned earlier, black and Hispanic students consistently score below the national norm in mathematics. It has been suggested that a student's *coursetaking history* in mathematics may have an effect on later achievement (Fennema 1977; Fox 1981; Lantz and Smith 1981). That is, if the student has not enrolled in or completed a mathematics course, it is unlikely that he or she would be able to score high on a test measuring the content taught in that course. In Marrett's study (1982) of forty-two high schools in thirty-six districts, it was found that black students comprised over half of the sample, but they were only one-third of the enrollees in Algebra II and were one-fourth of the enrollees in calculus. When she looked within racial category, Marrett found a greater concentration of black students in lower level courses. Nearly half of the black students, as compared with one-quarter of the white students, were in those courses. Moreover, black students represented a slightly smaller percentage of all mathematics enrollees than they did of the total student population. Her findings have been supported by a study by Jackson (1982) in which he used the "High School and Beyond" data bank and found that blacks were more likely to be enrolled in the lower level math courses.

There is evidence to suggest that there may be important *sex* differences within racial groups in achievement and enrollment (Matthews 1980; Thomas 1981). Marrett found that the differences between the sexes in enrollment did not tend to be large. Overall, it appeared that black males were more frequently found in lower level courses and less frequently in the higher ones. However, one should not exaggerate the trends for black females. Although more black females than males were in advanced courses, they were greatly outnumbered by white students of both sexes.

Age may be a factor in differences in outcomes, particularly after age nine. There is little empirical evidence to support differences at the younger ages. In one of the few studies of pre-school children, McNamee, Katz, and Bowman (1981) looked at teacher interactions with black low income children. Only one other study looked at students between kindergarten and fifth grades (e.g., Cuevas and Llabre 1981). We do know that after age nine, significant differences are found between minority and non-minority populations, and that the differences increase as the students grow older (Burton and Jones 1982).

There is some evidence to suggest that *mother's education and occupation* may be a better predictor of future student's success (e.g., performance) than father's influence. There is little empirical evidence to support this, however, because most of the findings reported for parental influence are perceptions of the students rather than actual observed or parent-reported influence. It is only from those studies of minority women scientists (Hall 1981; Kenschaft 1981) that we find mothers' attitudes about mathematics or attitudes about their children's ability to perform to be reported to have an influence. One should be cautious in concluding that

mothers have a larger influence. Due to the growing phenomenon of single head of household parents (usually the mother) of black and Hispanic students, reported mother's influence may be more a reflection of family patterns than direct mother intervention.

Influence of *language* has been studied in a few instances, and these are with Hispanic populations. Cuevas and Llabre (1981), MacCorquodale (1981) and Fernandez, et al. (1976) have reported the importance of the primary language spoken at home. Cuevas raises a very good point that a child whose native language is not English will have many problems with the language constructions used in the mathematics classroom.

Student influence. A number of student variables have been identified as having an influence on decisions to enroll in mathematics, among them are: *positive attitudes toward mathematics, stereotyping of mathematics, perceived utility of mathematics,* and *influence of significant others.* It has been reported that minority students do like mathematics, find it interesting, have little mathematics anxiety, and would like to take more mathematics (Matthews 1980; Nelson 1981; Rancifer 1978). There is conflicting evidence of whether or not minority students stereotype mathematics as a male domain (Matthews 1980; Rancifer 1978). Further, it has been suggested that minority students may hold racial stereotypes; that is, they may perceive mathematics to be a white domain (Hall 1981; Kenschaft 1981).

Minority students are less likely to understand how mathematics will be useful to them for their future jobs or post-secondary schooling (Lantz and Smith 1981; Matthews 1980). They also do not seem to understand how mathematics is used everyday and tend to think of mathematics only as something one does with a pencil and paper in the classroom.

Teachers and counselors have strong positive effects on students' attitudes toward mathematics. Minority students have indicated that they like a teacher who will work with them, will give them extra help, will explain things carefully, and who will encourage them in their studies (Matthews 1980). Early tracking of minority students into vocational remedial programs has been a continual problem, and counselors are particularly potent influences on fifth and sixth graders when decisions are made about course selection for the junior high or middle school years (Jackson 1982; Marrett 1981).

School influence. There are a few school-related variables that have been identified as having an effect on students' participation in learning of mathematics: climate, organization, racial composition, and personnel. Much of this evidence, however, is not empirical but qualitative.

Jackson (1982) has found that *school climate* variables such as discipline and attendance have an effect on black students' mathematics achievement. He found that the students were more likely to have lower mathematics achievement scores in schools in which there were attendance or discipline problems.

There is some evidence to suggest that the *organization of the school* may have an effect on black students' performance in mathematics. It has been found that in those schools in which more advanced courses are being offered, a small percentage of black students will be found to be enrolled (Jackson 1982; Marrett 1981). Minority women scientists have reported that the sequence and prerequisites for the mathematics courses as well as their placement in curriculum and the class size affected their decisions to continue to enroll (Hall 1981; Malcom, Hall, and Brown 1976). That is, a small class size, a placement in a more advanced track, and an early start on fulfilling the prerequisites for the more advanced courses were found to be important factors in the minority women scientists' education.

The discussions of the *resources of the schools* have been included in most of the studies. These descriptions are usually very brief, however, and only the inferences that the predominantly minority schools usually have poor facilities and materials available can be drawn. Overcrowding, use of trailers or other temporary classroom buildings, outdated materials, and lack of audio-visual and other supplemental materials are some of the problems cited (Malcom, Hall, and Brown 1976; Matthews 1980).

There is some evidence to suggest that the *racial composition of the school* may be related to the other school variables as well as the student outcomes. As mentioned earlier, both Mar-

rett (1982) and Jackson (1982) have found that the larger the black student population, the more likely that lower level mathematics courses are offered and the more likely that black students will be found enrolled in those lower level classes. The inverse has been found to be true also. The greater the white student population of the school, the more likely the school offers the more advanced mathematics courses and the more likely the white students will be found to be enrolled in these classes. Again, based on the reflections of the minority women scientists, the racial composition of the faculty seems to have an effect on students' attitudes and aspirations (Hall 1981; Kenschaft 1981). More often than not, these women cited black teachers, counselors, and administrators who were inspirational and influential in decisions to continue in a mathematics- or science-related career.

Although each researcher discusses the importance of looking at *personnel,* there has been little research conducted. In one study, student-teacher interactions were observed at the preschool level and it was reported that black students were more likely to be asked nonmathematics-related questions (McNamee, Katz, and Bowman 1981). Matthews (1980) found in her study that black students were reported to prefer a mathematics teacher who would give them a second chance to pass the test, who would stay and work with them after school, and who would follow the textbook page by page. This was contrasted with their white counterparts who reported to prefer a mathematics teacher who was warm and friendly, and young (under 35).

Recommendations

Future research. There appear to be a few assumptions that have been operating throughout these studies. The first assumption is that there is homogeneous performance within racial groups. It has been assumed that all students within a racial group are alike, and researchers have been less likely to consider differences in geographic location, socioeconomic status, as well as other background characteristics that might effect the profile of minority students. Not only are there difficulties in using gross classifications within a racial group, there are also problems when gross classifications are used in multi-racial comparisons. Just as pooling across racial groups may obscure important within-group differences, pooling across ethnic groups may mask important between-group differences.

A second assumption is that since all underrepresented groups share similar goals, they can be studied similarly. For example, a very fine model has been developed to study "women and mathematics" (e.g., Meece, Parsons, Kaczala, Goff, and Fetterman 1982). Several researchers have applied this model to studies of "minorities and mathematics." Moreover, we sometimes find the same instruments developed to measure sex-related differences in mathematics used to measure race-related differences. Given the language and cultural differences among racial groups and their effect on mathematics learning (e.g., Cuevas 1982), it may be that these models and instruments will have to be revised and new ones developed.

A third set of assumptions is that: (1) There exists consistency in what is learned in a mathematics class, at a course-specific level; (2) this knowledge can be measured objectively; and (3) comparisons can then be made across schools and across the nation. Many of us who have spent time in a poor, predominantly minority school know what those students are or are not learning in mathematics. Just as the "women and mathematics" researchers have found that looking solely at test scores sheds little light on understanding why there are differences, those engaged in research on "minorities and mathematics" must learn also. Researchers must spend more time in the classrooms if they hope to understand the processes of learning.

In the majority of the studies reviewed, the researchers collected cross-sectional data. Apparently, there is an assumption that students do not change over time, either in achievement or in attitudes. As suggested by Bloom (1980), both achievement and attitude are alterable variables that not only *can* be changed but in fact *do* change. Although researchers are more likely to take repeated measures of achievement over time, measures of attitude are often col-

lected only once. Since the primary purpose of research on minorities and mathematics is (or, if not, should be) to improve learning as well as to encourage participation, it becomes imperative that there be accurate measures of changes.

I believe that the instructional demands within each school setting are unique. Certainly, areas of commonality exist in teaching strategies and curriculum designs, but the differences between schools and between classrooms within schools may supersede the shared characteristics. This is not to say that we cannot learn from studies conducted within different school settings. But rather, I suggest that new questions should be raised as we search for a better understanding of those differences—questions which address not only what is being taught or how much the students have learned, but how they were taught, when they were taught, and by whom they were taught.

Rarely have minorities been involved as researchers in studies of minorities. Those that have known problems from within can bring sensitivity to the issues that others may lack. The importance of including minority researchers is paramount. I would hope that through such joint ventures, findings from the studies will more likely affect policy as it, in turn, affects the groups which the researchers represent.

I see a need to conduct small case studies of minority mathematics students. I suggest that these microstudies should include both quantitative and qualitative analyses. A concerted effort should be made to consider other background characteristics besides race in these studies. I also suggest that the length of time of these studies is not nearly so important as the quality of that time. Of equal importance is that the instruments are culturally validated. I would like to encourage educators, particularly mathematics educators, to become more actively involved in this research. I would like also to see more minority educational researchers taking an interest in this area.

Strategies for change. And so it seems that we do have a considerable amount of evidence that could suggest some strategy or intervention. For my discussion of "where do we go from here?" I am going to borrow from a speech given by Dennis Prager (1982) of the Office of Science and Technology Policy. In his presentation, a four-pronged strategy was outlined to increase the participation of underrepresented groups in the sciences. I have adapted this strategy for our own discussion.

1. *Identify goals.* The primary, if not singular, goal for us should be to insure equal access and equal opportunity for blacks to learn mathematics.

2. *Identify barriers.* In order to meet this goal, we must first understand the barriers which stand in our way. Research evidence suggests that many of these barriers may rest in the affective domain. Students are less likely to perceive mathematics as useful for their future jobs; parents, teachers, and counselors often do not support and encourage the students in their study of mathematics; and students tend to stereotype mathematics as a male domain. School variables (e.g., school climate, racial composition, personnel) can also be important barriers.

3. *Identify alternative approaches.* Having identified the barriers, we must then develop strategies for removing them. If we are unable to, we must then find ways to get around, or over, or through those barriers. This is where we need to begin to think about intervention and prevention. The intervention or prevention strategies we develop must be based on research of *successful* students. Too much of what has been studied previously has been of black students who were unsuccessful.

4. *Identify appropriate roles for the institutions.* There are a number of institutions that our black youth will encounter, and these institutions have an effect on the learning of mathematics. In schools, students are taught mathematics. Black students are not learning mathematics in the mathematics classrooms. In jobs, mathematics is needed for everyday operations. A mathematics background is required to complete many tasks successfully. If our black youth did not learn the mathematics in the classroom they are even less likely to receive on-the-job training to remediate these skills. Our government funds many of the research studies as well as many of the intervention programs. Blacks must make sure that they are studied, involved,

and represented in a fair and appropriate manner.

In conclusion, I believe that we can learn from research on blacks and mathematics. I believe that we can learn why there are differences. I believe that we can learn under what circumstances there are no differences. And I believe that we can learn how to decrease differences where they do exist.

It will require hard work. It will require courage. It will require dedication. It will require time. It will require money. A very wise person once wrote, "We have the power to determine who will be equal, who will be more equal, and even who will survive, simply by determining whom we shall educate." My challenge to you is to begin, and for some to continue, to investigate *how* to educate minorities in mathematics so that they might have an equal chance.

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BLACKS AND WOMEN IN MATHEMATICS

Patricia C. Kenschaft

This paper was presented at the Science and Technology Seminar sponsored by the Office of Special Programs, The City University of New York, on 3 March 1983.

Mathematical folklore claims that Albert Einstein once said that genius is ten percent inspiration and ninety percent perspiration. Dean Harris indicated that he saw the theme in my article about black women in mathematics as the *struggle* they all had to become mathematicians. He pointed out that this was true whether they were raised by a mother who was a waitress or by married parents where the father was a college president. It is true of whites and males, too. You do not get a good job in a technical field without working hard.

In this paper I will explore why anyone would want to struggle to learn mathematics, what is involved, and who has done it. The 1970 census included over a thousand black women who identified themselves as "mathematicians." I am sure there were far more black men then, and far, far more of both sexes by now. Still, according to Arthur Jones, a black man high in the hierarchy of the National Science Foundation, only about .7 of one percent of the Americans who have earned doctorates in mathematics are black. According to my research, there are thirty-three American black women who have earned doctorates in mathematics, and I calculate there are about one hundred and twenty men. Of the 396 Americans who earned doctorates in mathematics in the United States in the 1984-85 school year, four were black men and one was a black women. The first half of my talk will be telling you about some of these people. My information comes primarily from my own communications with them and their families.

In 1925, Elbert Cox became the first black person to get a doctorate in mathematics. His degree was granted from Cornell University and he spent most of his career at Howard University. He supervised more master's degree theses at that institution than any other person to this time.

David Blackwell is a professor of statistics at the University of California at Berkeley, which has sometimes been rated as *the* leading mathematics university in the country. He received his doctorate at the age of twenty-two from the University of Illinois in 1941. Later that year he became a visiting post-doctoral fellow at the Institute for Advanced Study in Princeton, the first black person to become professionally associated with that institution. Paul Robeson had applied to be a student there and had been turned down solely on the grounds of race. At that time members of the Institute were automatically fellows of the University, and thus Blackwell was listed in its bulletin as such. This caused considerable stir because there had never even been a black student at the University. Since the Institute was occupying University buildings, the president of the University wrote to the director of the Institute that the Institute was abusing the University's hospitality by admitting a black to its ranks.

Blackwell became the first black in any field to be elected to the National Academy of Sciences, the highest honor that can be bestowed on an American in mathematics or science. He delivered an invited address to the International Congress of Mathematicians in Amsterdam in 1954, was vice president of the American Mathematical Society, and was also president of the Institute of Mathematical Statistics. For several years he was department head at Howard University, and as such had a tremendous impact on younger black mathematicians.

J. Ernest Wilkins, Jr. was born in Chicago in 1923 and at the age of thirteen became a student of the University of Chicago. He received his B.S., M.S., and Ph.D. degrees from that university when he was sixteen, seventeen, and nineteen, respectively, and then he, too, spent a year at the Institute for Advanced Study in Princeton. Later he earned a bachelor's and master's degree in mechanical engineering from New York University in 1957 and 1960, respectively. He has held many research positions, both academic and non-academic. He has had more than seventy research papers published, in such varied fields as applied mathematics

and mechanical and nuclear engineering; he has also written over twenty unpublished but unclassified Atomic Energy Commission reports. He recently retired from his position as deputy general manager of EG&G in Idaho Falls.

In 1956 Wilkins was elected a fellow of the American Association for the Advancement of Science, and in 1976 he received the honor of becoming a member of the National Academy of Engineering. He was elected a fellow of the American Nuclear Society in 1964 and has held numerous offices in that organization, most notably the presidency in 1974-75. He was chairman of the Army Science Board from 1978 to 1981 and was awarded the Outstanding Civilian Service Medal by the Army in 1980. He has served on many prestigious boards including the Regents Council of New York State from 1953 to 1960 and the College Entrance Examination Board Advisory Committee on Mathematics Achievement Tests from 1972 to 1974.

Albert Turner Bharucha-Reid also had more than seventy mathematical publications. His research papers range widely over such fields as mathematical biology, theoretical probability, and functional analysis. He was born in 1927, the son of a member of the faculty of Hampton Institute; his father taught electricity there. His entire precollege education was in the laboratory schools of the college. He grew up in a home visited by some of the leading intellectuals of the day, including Paul Robeson and W. E. B. DuBois. His great uncle, T. W. Turner, was a professor of plant physiology at Hampton Institute and he had a great impact on his namesake; earlier he had been the second black person to earn a doctorate in the biological sciences.

Bharucha-Reid received his bachelor's degree from Iowa State University in 1949 where he completed a double major in biology and mathematics. His first research paper was written while he was a senior; it was a kinetics problem about diffusion in a cell. At first he regarded mathematics as a tool for serious research in biology, but eventually he became interested in mathematics for its own sake. After one year of graduate study at Iowa State, he became a research assistant at the University of Chicago for three years. During this time he wrote eight more research papers, primarily in mathematics and theoretical physics, but they all involved some aspect of biology. He laughed as he told me, "Then I decided there wasn't much point in my getting a doctorate. I *could* have put the papers together for a thesis—they did permit many different topics to be collected together to make a dissertation—but it didn't seem worth the work."

Bharucha-Reid held positions at Columbia University, the University of California, and the University of Oregon. One year he was a fellow of the Mathematical Institute of the Polish Academy of Sciences in Wroclaw, doing research in abstract probability with one of the leading mathematicians of the world. He gave invited lectures at universities throughout the United States, Canada, Europe, and India, and in 1963-64 he was a visiting professor at the Institute of Mathematical Sciences in Madras, India.

He was professor of mathematics at Wayne State University from 1965 to 1981 and director of the Center for Research in Probability there from 1967 to 1981. During this time he was the dissertation supervisor of thirteen Ph.D. students, one of whom was black. From 1976 to 1981 he was associate provost and dean of the graduate school at Wayne State. Then he became professor of mathematics at the Georgia Institute of Technology and in 1983 distinguished professor of mathematical sciences at Atlanta University. In 1982 he taught a calculus course in which there were 180 students. About twenty-five of them were black. "That's far more blacks than I ever taught before!" he told me.

Bharucha-Reid was co-senior editor of the *Journal of Integral Equations*, and associate editor of three other international mathematics journals, the *Bulletin of Mathematical Biology*, *Nonlinear Analysis*, and *Stochastic Analysis and Applications*. From 1972 to 1976 he was vice president of the International Association of Mathematical Geology. He died in February, 1985 shortly after being elected to the Council of the American Mathematical Society.

In 1949 Marjorie Lee Browne and Evelyn Boyd Granville became the first black women to receive doctorates in mathematics. Browne was born in Nashville in 1914, the daughter of a railway postal clerk. Her mother died shortly after her birth, but her stepmother taught her a-

rithmetic and reading before she began school. Her father had attended college for two years and was a "whiz" at mental arithmetic. He avidly shared his mathematical interests with both his children up through their college mathematics majors, learning from them as they progressed. He finally gave up following his daughter when she approached the border of mathematical research.

Browne majored in mathematics at Howard University and sang in its choir. Afterward she taught first in a private secondary school and then in a small college. When she visited her parents, she talked with a neighbor who told her that blacks could go to the University of Michigan and that the fees were not too high. She began attending during the summers, and finally won a fellowship to complete her doctoral dissertation full time.

After she completed her Ph.D. there in 1949, she joined the faculty of North Carolina Central University where she remained until her retirement thirty years later. During twenty-five of those years she was the only member of the faculty with a doctorate in mathematics. She taught fifteen hours a week of both graduate and undergraduate courses, and was department head for most of the time. With her leadership the University won the first grant to finance a summer institute for high school teachers at a traditionally black institution, and she ran these summer institutes for many years. Browne wrote four courses of study for these programs. She also won a grant in 1960 for a computer and then supervised its installation at the University.

She told me, "I always, always, always loved mathematics. I can do it alone. I do have plenty of friends, and I talk with them for hours at a time. But I also like to be alone, and mathematics is something I can do completely alone. If I had my life to live again, I wouldn't do anything else. I love mathematics." She died in 1979, less than two weeks after this interview. She had a tremendous effect on me; her kindness and the timing of that interview led me into the continuing pattern of asking mathematicians about their lives.

Evelyn Boyd Granville was born ten years after Browne in 1924 and they never met. Her earliest years were spent in the apartment building where her father was caretaker, but after he separated from her mother, she was supported by her mother's modest but steady civil service job. Her mother's twin sister, who never married, was like a second mother to her and contributed financially to her education. Like Browne, Granville was the second of two children. But unlike Browne she did not have any great advantages at home except people who believed in her and the importance of an education.

She credits her public school teachers with making it possible for her to become an outstanding mathematician. She attended Dunbar High School in Washington, D.C., an excellent segregated academic high school. There she studied under people educated in mathematics at Yale University and the University of Pennsylvania. These teachers encouraged their best students to go to the best colleges, and she was valedictorian. She went to Smith College. Her first year was financed almost entirely by her mother and her aunt, but then, fortunately, she won some scholarships that supplemented her own earnings and their contributions.

After graduating summa cum laude in 1945, she was awarded fellowships for graduate work at Yale University. Although she did not realize it, this was an incredible achievement for a black or a woman at that time. After receiving her doctorate in 1949, she spent one year of postdoctoral work at New York University. In 1950 she searched for an academic position. I discovered in 1980 that when she interviewed for one job in the New York area, they merely laughed at her application when they discovered that she was black, despite the fact that she had earned her doctorate at Yale University under Einar Hille, a leading mathematician of the time and former president of the American Mathematical Society.

She taught at Fisk University for two years where she was a role model for other young black women who later earned doctorates in mathematics. Then she spent fifteen years in government and industry. The first three years were with Diamond Ordinance Fuze Laboratories, analyzing mathematical problems arising in the development of missile fuzes. Then she spent four years at IBM working on the formulation of orbit computations and computer pro-

cedures for Project Vanguard and Project Mercury. The remaining years were spent with three different employers as a research specialist in celestial mechanics, trajectory and orbit computations, numerical analysis, and digital computer techniques.

In 1967 she joined the faculty of California State College in Los Angeles where she became a full professor. There she has served on numerous panels and advisory committees and coauthored a book, *Theory and application of mathematics for teachers.* In 1984 she "retired" to start a chicken farm with her husband and to join the full time faculty of Texas College in Tyler, Texas. She wrote to me that she feels "blessed" by a " very rich life," and added, "I always smile when I hear that women cannot do mathematics."

The other two women included in this slide show were "found" by Lee Lorch, a white male who lost four college teaching jobs in this country because of his efforts to help blacks. The first was here in New York because of his efforts to integrate Stuyvesant Town, the fine housing development in which he was living. A year after he left he was fired by Pennsylvania State because he and his wife permitted a black family to live in their apartment while they lived in Pennsylvania. Five years later he was dismissed from his job as department head at Fisk University in a vote mostly along racial lines because he had attempted to register his daughter in their neighborhood school in the month following the 1954 Supreme Court School Desegregation Decision. Ironically, she was never admitted to the school across the street where her friends went, but was always bused across Nashville to a "white" school while her family lived there. Finally Lorch lost his job at Philander Smith College in Little Rock, Arkansas after his wife rescued a black student who expected to integrate the high school there and instead found herself facing an angry mob. Unable to find another job in this country, he has taught in Canada for over a quarter of a century. There he has been elected to the Royal Society of Canada, the counterpart of the Academy of Sciences in the United States. Nevertheless, he was twice elected to the Council of the American Mathematical Society and chaired a National Science Foundation committee that distributed money to research mathematicians in the United States. He has toured Europe and Africa in response to widespread invitations to speak about mathematics, and in 1980 he was sent by the United States to East Germany for a month.

There are only four graduates of Fisk University with doctorates in mathematics and only one graduate of Philander Smith College. All five studied under Lorch. He observed to me wistfully once that he had been able to prepare about one black a year for a doctorate for each year he spent at Fisk University. If he had been allowed to spend the past thirty years at Fisk, what would be the participation of blacks in the national mathematical community by now?

One of the three black women with a Ph.D. in mathematics who studied under Lorch at Fisk is now department head at Spellman College. The other two are Vivienne Malone Mayes and Gloria Conyers Hewitt. Mayes was an upperclassman during Lorch's first two years at Fisk and remained with him for two years of graduate work. By the time she received her M.A. from Fisk in 1954, he had stimulated her to want to do research. Later she enrolled in a doctoral program at the University of Texas at Austin.

In her first course there she was the only black and the only woman. The others completely ignored her. It seemed to her that conversations between classmates about mathematics that went on before class quickly ended when it appeared that she was listening. Because of her race, she could not become a teaching assistant, nor could she discuss mathematics with her adviser and fellow students at nearby Hilsberg's Cafe. She joined the picket lines, and only after the law was changed to require the cafe to serve blacks did she discover that women rarely entered, no matter what their race.

These stories are especially remarkable to those of us who know Vivienne's delightful personality. She is often invited to speak and has been elected to several offices in professional organizations. Since receiving her doctorate in 1966, she has taught at Baylor University in her home town of Waco, Texas, a predominately white institution. In 1971 the Student Congress elected her the Outstanding Faculty Member of the Year. She told me that for many years "the feds" visited Baylor regularly to be sure she was being treated fairly and her promotions came

on time. Under the Reagan administration the visits stopped and her life has become more difficult.

Gloria Conyers Hewitt was a freshman and sophomore during Lorch's last two years at Fisk. When she was a senior, she was startled to receive invitations from two graduate schools on the basis of his recommendations. She told me that until they came, she had never considered going to graduate school. Furthermore, she did not know that Lorch considered her research material until she heard of his recommendation. She accepted the invitation from the University of Washington, and her experience there was in startling contrast to Mayes' at Texas.

She soon discovered that her mathematical education had essentially ended with Lorch's departure two years earlier. She was shy about asking for help, but the other graduate students, all of whom were white males, would come to her office and volunteer to explain "interesting" problems. Once one said, "The trouble with you is that you don't understand trigonometry. I am going to teach it to you." Two hours later she understood trigonometry.

Eventually, of course, she felt free to ask them questions whenever she was puzzled. She felt at home socially in the group. When they went to a bar in the evening, she went too. If they went to the best restaurant in Seattle, she came along. Despite her inferior preparation, she completed her doctorate in only five years, ending in 1962. She joined the faculty of the University of Montana at Missoula where she has been a full professor since 1973. She has held numerous leadership positions in professional organizations and has had two postdoctoral leaves to pursue her research.

My own most extensive research has concerned American black women with doctorates in mathematics. Most of these women received their elementary and secondary educations in segregated Southern schools, many after the 1954 Supreme Court decision which declared that such schools were unconstitutional. All but one grew up attending black schools; the other grew up in an essentially all white community. The only one who is a product of the New York City Public Schools told me it makes her very angry when people deflect the discussion of how to provide quality education to black youngsters with "side issues" such as busing. As I look at my own integrated town—about half the students in the Montclair, New Jersey school system are black—I am acutely aware of the fact that no black woman with a doctorate in mathematics is the product of an integrated school system. Several of the women commented that although the Southern segregated schools were inferior mathematically, they were more inspiring to young black students than other schools because of their role models, their faith that blacks *can* learn, and the value they place on education. This last is too often lacking in Northern schools that blacks attend. You have here something important, and I expect great results from your program.

I know the careers of the parents who supported half of these women when they were children. Two of the fathers were presidents of black colleges, one was a college professor, and one a high school teacher. Six of the fathers had steady jobs outside education — one was a real estate broker, one a railway postal clerk, one a civil service political aide, one a barber, one a letter carrier, and one an unskilled laborer. The three mothers who supported their daughters included one college professor, one civil service employee in Washington, and one restaurant worker. The waitress mother, incidentally, raised alone the only one who did not grow up attending black schools.

Most of the women on my list married at least once and a majority are mothers. There is a false belief in the white community that educated women with good careers are not in demand as wives. My observations suggest that while the demand is small, the supply is smaller—and nobody needs very many husbands. There are rumors that black men are more broad-minded than white men about permitting their wives to make lots of money, but I can see no social reason why any girl should not go into a mathematics-based field. You meet *lots* of nice men in these fields, and you have plenty to talk about with them!

About half of the black women with doctorates in mathematics received their undergradu-

ate education in a traditionally black institution, and the others in predominantly white colleges. Currently eleven teach mathematics in a traditionally black institution and six at other institutions of higher education. The others include four mathematicians in industry or government, one full time mother, a high-level administrator of education in Washington, D.C., and the dean of students at the Massachusetts Institute of Technology.

Clearly blacks have made a significant contribution to the mathematical life of this country. Just as clearly there remains a long way to go before equity is attained. If the recent momentum is not used, and the existing role models are not publicized, the number of blacks in mathematics is sufficiently small that their participation may experience a decline similar to that of women earlier in this century.

Dean Joseph Harris asked me to give some information about women in mathematics, and this is a good transition. The first American to get a doctorate in mathematics did so in 1864 at Yale University. The first American woman received her doctorate in mathematics from Columbia in 1886, only twenty-two years later than the first man. So women broke into the doctoral mathematics research community almost forty years earlier than the first black man.

Women were active in mathematics around the turn of this century. Before 1940 approximately 14 percent of the doctorates in mathematics awarded in this country went to women, but that percentage dropped to 5 percent in the 1950s and 6 percent in the 1960s. During the 1970s it grew unevenly and in 1981 again achieved the pre-1940 level. However, data from the American Mathematical Society survey—the best we can get—indicates that last year only 8.0% of the faculty of doctoral-granting mathematics departments in this country were women, compared to 8.7% the year before and 12% in 1981. The same survey indicates that over 70% of the men in the mathematics departments of these universities have tenure, but less than 40% of the women. Some of this may be due to the fact that the women are younger. In any case, women have a good-sized toe-hold in numbers, and we are not going to be shoved out of the mathematical community easily.

However, I fear that if the achievements of blacks are not recognized and used as role models, their absolute numbers are small enough that they may fall far behind their previous achievements, as women did in the middle of this century. That is why I am here today. I will leave a copy of this talk with Dean Harris, and I will be glad to supply copies of my slides of black mathematicians to anyone who requests them. I hope people will use the slides and my research to tell black students that they are not alone.

In 1883 the feeling of isolation for women was even greater than that of blacks now, since no English-speaking woman (and only one other) had ever earned a doctorate in mathematics. Charlotte Scott was teaching and studying at Girton College and in 1885 would become the first such recipient. Girton was the first college in England to admit women; it had opened in 1869. In 1872, the three members of the first class had taken their undergraduate final exams at Cambridge University and passed. Their names were Woodhead, Cook, and Lumsden, and they were known as the Girton Pioneers. In the late 1870s, when Charlotte Scott was an undergraduate at Girton, the students would sit around in the dark evenings and sing inspiring songs. I know the words of one, and I want to share them with you to give you the feeling of those times.

Whenever we go forward
A hard exam to try
Their memory goes before us
To raise our courage high.
They made old Cambridge wonder;
Then let us give three cheers
For Woodhead, Cook, and Lumsden
The Girton Pioneers.

There is a second and a third verse of similar spirit, and this is the fourth:

And when the goal is won, girls,
And women get degrees
We'll cry, "Long live the three girls,
Who showed the way to these!"
Who showed the way we follow
Who knew no doubts or fears—
Our Woodhead, Cook, and Lumsden
The Girton Pioneers.

Women did not get degrees from Cambridge University until 1949. It was a long battle. But meanwhile, after doing well on the Cambridge exams, Charlotte Scott and other women took similar exams at the University of London, which did begin granting "external" degrees to women in 1876. In 1880 Scott earned her bachelor's degree and in 1885 her doctorate. Since women could not at that time get suitable jobs in Europe, she crossed the Atlantic Ocean and in 1885 became a member of the first faculty of Bryn Mawr College. For forty years she was department head there, and she was one of the first people to do mathematical research in this country whose work was widely quoted in Europe. She was on the first Council of the American Mathematical Society, and she was its vice president in 1905. I have located almost forty mathematical publications of hers.

While Charlotte Scott provided leadership, women took a strong role in the American mathematical community. But when she died she was essentially forgotten, and the percentage participation of women dropped to about one-third of its former level. When Mary Gray was elected vice president of the American Mathematical Society in 1976, she thought she was the first woman to hold that position.

Julia Robinson (1919-1985) was the president of the Society in 1983 and 1984, the first woman to be president. A world famous mathematician, she had no faculty position until her husband retired. Then the University of California at Berkeley hired her! Indeed in 1970 there had never been a woman at any of the five leading mathematics institutions in the United States, although David Blackwell had been on Berkeley's faculty for a long time. Now there are two women on those five faculties.

At the lower levels we are much better represented. According to a survey done in 1976, slightly over half of the membership of the National Council of Teachers of Mathematics were women. However, only about one-quarter of the math teachers in this country belong to the Council, so we do not know whether that reflects the proportion of math teachers in this country, or whether women are more likely to be joiners. About 12 percent of the American Mathematical Society membership —the organization to promote research mathematics—are female.

Why are there not more women in mathematics? It is not lack of interest. Studies repeatedly show that boys and girls have approximately the same interest in mathematics, unlike science, where boys seem more interested, and languages, where girls lead. However, it seems to me that the small proportion of women at the top of the mathematical community is not surprising when we consider three necessary qualities a woman must have to get a doctorate in mathematics.

First, she must be better than her typical male competitor. In a few minutes I will consider at greater length what makes a person good at mathematics.

Second, she must believe that women can achieve. That is, she must be an ardent feminist, whether or not she admits it.

And, at the same time, she must be able to get along with men because they will essentially provide all the friendship she will get when she is doing graduate work in mathematics. The combination of being an ardent feminist and being able to thrive with only male friends during one's work week is not a common combination.

I think that there are similar prerequisites these days for blacks in mathematics. A superachiever *must* believe that blacks can excel in mathematics *and* he or she must also be willing and able to spend the work week almost entirely with whites. About nine-tenths of the doctoral mathematicians in this country are white males; those of us clawing for a place in the mathematical community have to be able to tolerate that fact. However, I believe that a woman or a black who feels comfortable with his or her race and sex is much better able to reach out to any others as people and build friendships based on a common interest in mathematics. I believe we can help our female and black students to be proud of their heritage and yet sufficiently aware of the special obstacles they face to keep struggling without taking each defeat as a personal failure. One must struggle as if one's success depended on one's own efforts, but one must view one's failures—and some failures are inevitable for everyone, regardless of how fortunate or unfortunate we are—*not* as evidence of one's own inability to achieve.

How do we help students become good at mathematics? I was recently employed by the Montclair Public School system to locate math programs suitable for their gifted middle school students. I could find only one, the former Columbia mathematics program, now modified and called the Unified Math Program and published by Addison-Wesley Publishing Company. It is a six-year program designed to take students from the seventh through twelfth grade and introduce them to serious modern mathematics. Montclair has been using this program for years, and my own two children took the first four years and were then well prepared to begin calculus and do very well.

For many years Montclair required that students score about the 90th percentile on their sixth grade Iowa tests to enter the program. In an effort to get more blacks into the program the entering score was lowered to the 85th percentile last year, and there was much protest from the teachers. However, I discovered in my investigations recently that the program is used in the Chicago public schools, which are not too different from the New York public schools. They call a student "gifted" if he or she scores in the 72d percentile in the sixth grade standardized tests, and such students, well taught, can handle the seventh and eighth grade Unified programs supplemented with some remedial math. They then do very well in traditional high school mathematics. In my ideal world, they would then be able to take the ninth and tenth grade Unified programs instead of the traditional subjects. Incidentally, I should say here that the "topical" or "unified" program now offered under the New York State regents is a *much* simplified form of the Addison-Wesley program, published by at least three other publishers. I have not looked carefully at it, but I have been told it is not nearly as challenging as the Addison-Wesley program.

Chicago has a training program for its teachers, taking them in an intensive two-week program through the seventh grade program and through the eighth grade program in another two-week program. Permitting elementary teachers to take this inservice training program has obvious advantages, and I would suggest that you contact Yossel Naiman, the Chicago director of programs for the gifted, at 312-763-8252. I am sure he would be as helpful to you as he was to me. The college professor who taught the Chicago teachers told me that she believes that there is a Piaget-type readiness for abstract thinking in the middle school years. It is like languages. You can learn it later, but it is harder and you have an accent. She also feels that the seventh and eighth grade Unified programs are excellent teacher training courses for elementary school teachers, and Chicago tries to get as many as possible of these to take the program. If you can get your own faculty to be enthusiastic about the program, I am sure you can have impressive results, too.

How do we help people—faculty and students—to become enthusiastic about a mathematics program? First, they must have some understanding of what mathematics is. One black mathematician told me that the reason it is so difficult to get blacks to study mathematics is that they think higher mathematics consists of memorizing larger and larger multiplication tables. I fear this misperception is widespread.

What is mathematics? My current definition is that mathematics is the exploration of patterns, the communication of patterns through symbols and writing, and the use of such patterns

to solve problems. An exciting aspect of mathematics is seeing the same pattern appear in many different applications.

There are many niches in our economy for people who like exploring patterns and applying them. There is a desperate shortage of mathematics teachers, even in the pleasant suburbs where I live, because people trained in mathematics are finding jobs in computers, in business, in insurance companies, and in statistical work—especially in the environmental and pharmaceutical fields. The graduates of our department this year typically started with salaries over $20,000, no matter what their undergraduate record, and no matter what their race or sex. Mathematically-trained people are rarely rich. But they are not poor, either. And—possibly this is the most important thing to get across to the students—they *like* their work. Of the first forty members of the class of 1982 in our department to answer a survey that I took following their graduation, only one checked that she liked her work "not much." Six others said they liked their work "enough," and the rest checked either "very much" or "enormously."

I worry about the way other countries are teaching their young folk mathematics while we concentrate on "basic skills." Japan has twice as many engineers with half as many people. In 1981-82 a Japanese girl lived in my home from July to July under the auspices of the American Field Service and I had an opportunity to corroborate what the experts are reporting. She wanted to be a high school guidance counselor. But to get *into* any college in Japan she had to take an exam in calculus up through volumes of slicing. It is the standard eleventh grade curriculum in that country. To get out of the tenth grade in Japan, any student must take trigonometry and matrix theory—and they have the same proportion of students in tenth grade as we do. Nevertheless, this is the standard tenth grade curriculum even for people preparing to repair cars or cut hair. No wonder Toyota is the best selling car in the world!

Why is our entire country so far behind? Are our students less smart? I doubt it. I think we are expecting too little of them. Dean Harris is a nice guy and I think he can handle my picking on him a little. When he first telephoned me, he said that the students involved in your schools may not be doing as well as in some others but he thinks they have something to contribute. Well, I think that is a great understatement. I think some of your students are potentially important leaders of my country, and I want them to have a chance to become leaders. Unfortunately, it is all too human for teachers—I include myself—to observe with despair the poor preparation of their black students and conclude that they cannot learn. It is well verified that the teacher's perception of a student's ability affects how the student learns. Alas, it is also true that our culture teaches us that blacks and females do not learn mathematics as well as white males. Even though you and I do not believe this intellectually—even though we ourselves are black or female or both and clearly able to learn—we still carry the emotional baggage of our culture, and we plant it in our students, however much against our wills.

I think this unintentional imprinting of teachers at all levels, coupled with differing home and peer group environments, explains the difference of racial groups on such tests as the college boards. Recently, I conducted some investigations for my own town of Montclair, and the importance of a child's environment has seemed overwhelming. I hope you will pardon some personal examples. My own brother was diagnosed as mentally retarded when he was four, but my parents decided he "had something to contribute" anyway. My childhood and youth was spent watching my parents struggle to make something of his life, and now he sorts and delivers mail for a large company. He lives with my parents, helps with the housework, pays rent, but nevertheless saves money. He has saved so much that he now holds the complete mortgages on all three of his siblings' houses, including mine. My brother is now able to contribute because he himself was helped and educated appropriately for his ability.

However, if parents themselves were not given opportunities, and—even more important—hope, then they cannot help their children as much, and more falls to the schools. You and I are working with many students who have not had the opportunities at home that some students have, and this affects the way we look at our students. Some years ago I taught a summer program at Bloomfield College for black graduates of Newark high schools who had won scholarships for the following fall. I was to help them make the transition. These were

near the top of Newark's graduates. They worked reasonably hard and were smart enough for me to find the job very satisfying. Their ability and knowledge, of course, varied considerably. One day I worked individually with three of them and discovered that none of the three could find 25 percent of 40. The next evening at dinner, I told my family. My six-year-old daughter looked as surprised as the adults. "Why mother," she exclaimed. "Isn't that ten?" I wondered then, and still wonder, how those gifted young folk who have not been taught even the basics about percentages are going to compete with children such as mine, who feel comfortable with percentages in first grade and whose dinner table conversations often include discussions of mathematics. Is it any wonder that children with parents who did not even complete four years of high school mathematics are at a tremendous disadvantage?

Still, my survey of black women in mathematics shows that a mathematical home is not needed for a student to become a college professor. It is possible for the schools to teach mathematics. However, Arthur Jones, the black man at the National Science Foundation whom I quoted before, argues that the "back to basics" movement is hurting the prospects of blacks as a group in the mathematical fields. There is much too much emphasis on computational arithmetic—the type a hand calculator can do—and this is taking time from the teaching of patterns. Mathematics, the study of patterns, is very different from learning arithmetic tables. Those tables are important for learning mathematics and I used to drill my own children on them as we were driving. It is a good way to keep little kids quiet during a long car trip, and I wonder why more parents do not while away the time drilling arithmetic facts, multiplication tables, and simple mental equations. It is fun, and little kids love it. But it is not math.

As a mother who wanted very much for her children to enjoy mathematics, I also looked for opportunities to teach real mathematics to my children. One of my daughter's first memories is of the time she wanted to find the number of holes in the door of our clothes drier. She began to count, and I showed her how she could find the number by multiplying the number of rows by the number of holes in each row, plus a few left over. She was delighted. That was an adventure in seeing how the patterns she had learned in multiplication could be used to solve a problem. It was mathematics.

These examples from my own home life indicate the kinds of advantages that young folk who get scores of over 700 on the college boards have had. It is not just that they are smarter! They have been taught. And even Head Start programs get them later than a super-motivated mother like me will begin. I taught both my biological children about the commutative law of multiplication while changing a diaper. I have often thought what that revealed about my own priorities; for me a deep understanding of multiplication was more important than toilet training. After all, I figured, they would be toilet trained by the time they began college. One cannot be so sure about the commutative law of multiplication.

The point in telling you these personal experiences is to emphasize the idea that even if we ignore the unreliability of tests and the fact that students have "off" days, it is easy for me to believe that a student measuring in the 72d percentile without early training in mathematics may have just as much native ability as one in the 99th percentile who was taught mathematics as a pre-schooler. Your students do not just have something to contribute; many of them are gifted people whom *you* can enable to become leaders living highly satisfying lives.

The black women I have interviewed all had two things in common other than great ability and emotional stamina. They all had families who encouraged their academic aspirations. The father who was an unskilled laborer did not know mathematics, but he was glad his daughter wanted to learn it. Family support seems to be necessary to get a Ph.D. in mathematics, but it does not seem to be required to become a good secondary school teacher. What seems to be necessary for both levels of achievement is for a student to have had a good secondary school teacher who recognized the student's ability and told him or her that a career in mathematics would be worth the struggle. Almost every mathematician I have interviewed had this in common. Secondary school teachers hold the irreplaceable key.

The women I interviewed told me that they felt that race was their major disadvantage before college, and sex during college and afterward. But anyone, of any race or sex, who can

get a college degree in mathematics has made it to the middle class. They find agreeable, steady, and decently-paying jobs. And they like their work. It is worth the struggle. I congratulate you for your work and wish you the very best of success.

ADMINISTRATIVE AND CURRICULAR ISSUES IN MATHEMATICS EDUCATION OF MINORITIES AND WOMEN

Joan E. Coleman

In the face of growing national concern for the serious underrepresentation of minorities in the scientific and technical fields, a number of foundations, educational, industrial, and governmental institutions have initiated studies to determine the reasons underlying the low participation of minorities and women in these areas. In general these studies indicate that minority students tend to take fewer high school mathematics courses; achieve lower scores on mathematics-based aptitude and achievement tests; and are underrepresented in collegiate mathematics-based curricula (Morning et al. 1980). Consequently, they hold proportionally fewer academic and corporate positions in technical areas than their non-minority counterparts. Notably, mathematics has been identified as a critical filter that eliminates a significant portion of women and minorities from the fields of mathematics and science, which provide the underpinnings of industrial technology and defense and open the door to a variety of challenging and lucrative opportunities.

Educators, professional organizations, industry, and government generally agree that the present system of mathematics education needs improvement. Some have suggested major reform in the way the subject is taught, the number and kinds of courses offered, and in the training of mathematics teachers. However, student learning, related attitudes, and achievement in mathematics are, to a significant degree, dependent on the quality of instruction, the administrative priority, and the fiscal support given to mathematics at the local, state, and federal levels. This article will review and discuss major administrative and curricular issues in mathematics education and their impact on minority student learning.

Teacher Education

The responsibility for the preparation of elementary and secondary teachers is vested in university schools of education. Current teacher education curricula do not place significant emphasis on formal mathematics education. Frequently, survey-type courses such as "mathematics for teachers" are the only mathematics courses required. The lack of solid grounding in mathematical concepts and analysis often results in a fragmented or computation-focused approach to instruction. Further, current university teacher training curricula do not allow for the involvement of mathematics or scientific disciplines in the formal education, classroom observation, or certification of teachers. The absence of a joint effort between schools of education and mathematics/science departments inhibits the kind of orientation and sense of mathematics application that teachers could pass on to students. For example, an earlier study (Ernest 1976) found that 40 percent of the students in a mathematics course for prospective teachers at the University of California at Santa Barbara indicated either a negative or indifferent attitude toward mathematics. This finding is alarming in view of the fact that Ernest also found teacher attitude to be one of the most significant influences on student attitude toward mathematics.

There has been some suggestion that teacher education should be school-based. That is, the professional aspect of the training of teachers needs to be centered in the schools and controlled by them as a "technical training," comparable in some ways to industrial training. Here, the role of higher education in the education of teachers would be to provide a good general or liberal arts education in the first three years of college. School-based professional training would be offered in the fourth and possibly the fifth years. The former Department of Health, Education and Welfare Teacher Center program grants to establish experimental teacher learning centers (combining the traditional teacher college and the traditional liberal arts and science

college) addressed this concept. In many ways, the debate of university versus school versus community responsibility for teacher education is a moot one. It is more profitable to view the responsibility for teacher education as a part of the whole problem of school reform. In this context, the crucial question for mathematics education is "What is the nature of the partnership or coalition that can address the problem most effectively?" This, of course, is a question of policy.

Finally, there is some indication that professional teacher organizations have not supported state efforts to upgrade qualifications and requirements for entry into teacher education programs. A noticeable example is the state of New Jersey where the Department of Higher Education encountered considerable difficulty in raising the grade requirements to 2.5 (on a 4 point scale) for entry into teacher education programs in the state colleges. The Department experienced equal resistance in raising accreditation standards for teacher education programs in the state. The maintenance of such policies has had a negative impact on the quality of education and teaching in New Jersey.

Teacher Certification/Hiring

Current certification requirements for mathematics teachers are inadequate. In some states, certification requires little or no college mathematics; consequently large numbers of teachers with limited training are hired and constitute the pool of available manpower. In times of fiscal stringency, reassignment of such teachers to advanced courses is impossible without extensive staff development. In-service training, which might improve teacher competency and increase interest in mathematics, is also affected by limited funds and budget cutbacks.

The lack of mathematics competency among elementary and secondary teachers nationally raises serious questions about the validity of school-based training. Some form of university and industry linkage is necessary if both student instruction and the curriculum are to keep pace with the current technological knowledge. In many instances, the content of introductory college level calculus courses is more advanced than it was fifteen to twenty years ago. Consequently, students who take three years of college preparatory mathematics may meet entry and enrollment requirements for a university, but their conceptual competence falls short of what mathematics professors expect in classroom performance.

Curriculum and High School Graduation Requirements

There is general recognition that the secondary school mathematics curriculum is not keeping pace with new technology and should be updated and expanded. In most states and localities as little as one year of mathematics is required for high school graduation, and in many instances the course required is basic mathematics (arithmetic). Generally, advanced mathematics (and science) courses have no priority as basic requirements. Only one of the ten basic skill areas in mathematics cited by the National Council of Teachers of Mathematics, basic computation or arithmetic, is taught across the board in elementary and secondary schools. Further, instruction fails to concentrate on those aspects of mathematics which require reasoning; these include problem solving, probability, estimation, graphing, relations among variables, and data interpretation.

An important goal of the mathematics curriculum is to teach students to apply the mathematics they learn to new or different situations. Recently released results from the third mathematics assessment of the National Assessment of Educational Progress (NAEP) indicate that although students are learning many basic computational skills, they have difficulty applying these skills to solve even simple, non-routine problems. For example, more students were successful in solving simple textbook problems that could be solved by applying a single operation to the numbers given in the problem. However, any exercise that required students to do more than decide whether to add or subtract, multiply or divide, caused considerable difficulty.

They found that frequently students would attempt to apply a single mathematical operation to whatever numbers were given in a problem rather than analyzing the problem to decide how to solve it (NAEP 1980). These results strongly suggest that students have become accustomed to applying mathematical algorithms mechanically to problems without analyzing the problems or thinking about the reasonableness of their answers.

The National Council of Teachers of Mathematics has recommended that "school mathematics at all levels should be organized around the development of problem solving ability." It recommends further that computer literacy and calculator facility should be a part of the mathematics education of every student. A Council survey showed professional support for its call for a three year mathematics requirement for all high school graduates. Another survey recommended a requirement of at least two years of mathematics for non-college bound students.

Basic Competency Standards

Widespread concern about long term declines in students' mathematics (and other) proficiency test scores has led to the institution of minimum competency testing at various grade levels of elementary and secondary schools across the country. One result of this "back to basics" movement is that curricula and classroom instruction are often geared to proficiency tests. In many instances proficiency tests emphasize the most superficial aspects of learning in the content areas, focusing on the recall of facts and information as opposed to an understanding of conceptual schemes. This narrow focus tends to minimize (rather than maximize) learning by holding students to a norm and lowering teacher expectations of student achievement. When application and conceptual understanding are not emphasized, students cannot integrate and build on what they learn.

There is evidence that proficiency test results may not provide an adequate measure of achievement. In a cross cultural study, Brooks (1972) found that some students use memory rather than analysis to a greater extent than others in problem solving. These students may show achievement gains or basic skills in tests while, in fact, they are falling behind in areas requiring complex cognitive processes.

The aforementioned limited and outdated curricula and graduation requirements have produced a pool of "mathematically illiterate" graduates, many of whom are not prepared adequately even for technology-oriented training programs which could provide them with substantial careers. The issues of curriculum requirements and competency testing raise several questions. (1) How can the definition of basic mathematics competency be expanded to include more than the present minimal computational skills and with what objectives? (2) What is the optimum number of years of high school mathematics needed to prepare graduates (a) for college mathematics and science, (b) to cope with the increasingly technological developments of everyday life, and (c) for entry into technology-oriented labor markets?

These questions raise issues which may vary from state to state; nevertheless, they call for reassessment of existing practices and policy.

It is important that any final assessment of student performance be designed to measure the student's ability (1) to perform at or above the basic competency level (pre/post-testing), and (2) to demonstrate the broader cognitive skills required for subsequent learning. For example, he or she must demonstrate the ability to know, comprehend, apply, and analyze what he or she has "learned." It is here that the role of developmental (versus remedial) instruction becomes most important. There is sufficient data (Coleman 1982) to indicate that many underprepared students complete remedial courses in mathematics, reading, and writing successfully, yet fail to pass introductory courses in various disciplines. Analysis of student failure in these courses indicates the need for a kind of "transitional" instruction which bridges the gap between isolated skills and the successful application of these skills in more advanced courses. These findings reinforce the need for expansion of the high school basic arithmetic curriculum.

Problems in Instructional Methodology

Much of the literature and research in mathematics instruction addresses the mechanics of teaching but fails to examine the assumptions and philosophies which underlie the discipline of mathematics. Historically, mathematics has been molded by the need for solutions to problems. Techniques were developed in response to realistic situations. Accordingly, the essence of mathematics lies in the thought process involved in mathematics development, not in the algorithms developed. Many mathematics educators contend that in order to improve the quality of mathematics instruction it is necessary to reexamine and revise basic assumptions about the character of mathematics learning. Proposed revisions should focus on the nature of mathematics and the characteristics of the learner.

Most mathematics curricula are not designed to assist students in the process of mathematical inquiry. Generally, teachers proceed by determining the content to be taught and translating this into specific skills which, in turn, are organized into a hierarchy of traditionally sequenced, discrete instructional units. Thus, thousands of years of mathematical knowledge and development are reduced to a finite set of skills and rules which are imposed on students in the course of one or two semesters. This process is artificial in that it ignores completely the developmental nature of mathematics. Current course structures often present or combine topics which, historically, did not develop sequentially and therefore need not be so presented, especially if this presentation "overloads" the student. For example, in most course outlines the study of integers, a new number system, is followed closely by algebraic expressions which involves another new topic, literals (letters). Since algebraic manipulations developed independent of integer arithmetic, it can be taught initially using familiar whole numbers, thereby facilitating learning. Once the student masters the manipulation and understanding of literals, his learning can be reassessed in the context of integer arithmetic (Upchurch 1980).

The lack of sensitivity to the nature of mathematics is compounded by the lack of attention to a student's learning needs. The automatic "recipe" of content and sequence isolates the student from the learning process. He or she is treated as a "receiver" of information, yet is expected to incorporate these pieces of knowledge into a set of working mathematical foundations. Remedial mathematics courses, which are designed to correct "deficiencies," assume erroneously that, except for a gap here and there, the student in these courses thinks about mathematics in the same way as the student who has completed mathematics courses successfully. Teachers assume that their task is to supply the information unlearned or unsupplied in previous mathematics courses. Consequently, many women and minority students spend excessive time repeating entire mathematics courses unsuccessfully, without receiving the kind of diagnosis and instruction which would enable them to understand and develop mathematical thinking and "discovery."

At the undergraduate level, underpreparation in mathematics problem solving creates difficulties for minorities (and other students as well) on a par with writing and analytical reading preparation. Consequently, it is necessary to address this instructional need in a more comprehensive way than exists presently. The primary instructional offering for entering college students who lack three years of mathematics or whose SAT, achievement, or placement scores fall below an acceptable cutoff is, generally, a precalculus review course which covers in one semester (or quarter) a rapid review of the content of tenth, eleventh, and twelfth grade mathematics. Often this course is the only opportunity students have to qualify for enrollment in calculus. Many entering freshmen have not had one or two years of mathematics, yet they are forced to enroll in these review courses since they are, in most instances, the prerequisite for calculus. Often such courses "review" material that students have not seen or learned previously. Furthermore, given the rapid pace, the emphasis of necessity is on manipulation of formulae. It is a widespread perception of mathematics faculty that entering students are characteristically weak in problem solving, that is, the ability to comprehend a verbal statement of a technical problem, to formulate it in mathematical terms, and then find the solution. Hence even though many mathematics students may learn to solve systems of linear equations or manipulate trigonometric identities (the third step in the problem solving sequence), they tend

to bog down in the first two steps—understanding the problem and translating it into relevant formulae. This weakness in problem solving creates difficulties in all areas of mathematics, science, engineering, and quantitative social science. Essentially, we are finding that there is a gap between what we *require* and what we *expect* of students. That is, our college admissions requirements are not in line with expectations of classroom performance.

The College Board has addressed this issue in its report *Academic preparation for college: What students should know and be able to do.* The Board has, in fact, determined the basic academic competence students need for effective learning in mathematics (and other subject areas). The report points out that although basic academic competencies (defined as "developed abilities, and outcomes of learning discourse) are not always identified explicitly, spelling them out provides a way to tell students what is expected of them." The implication here is that knowledge of what is expected is crucial to effective learning. The College Board's outline of the special knowledge and abilities students need to perform effectively in college in each of the primary disciplines provides a useful strategy for higher educational institutions to reassess and integrate requirements and performance expectations.

The process of integrating new skills with prior knowledge requires much deliberate guidance. If one recognizes the belief that mathematical "knowing" does not happen spontaneously, it is necessary to pose new criteria for organizing and teaching mathematics courses. Also, mathematics content development should address the importance of mathematics to the student with problems that stem from the realities of the scientific, economic, social, and ecological world, proceeding from example to technique to principle—instead of the reverse, which is usually the case.

Student-Centered Affective Issues

The 1980 Ford Foundation reports on Women and Minorities in Mathematics present a detailed analysis of the attitudes, characteristics, achievement, career choices, and problems particular to these two groups.

Generally, female students demonstrate a preference for fields that lead to traditional "female" professions. Also, they tend to avoid fields requiring extensive application of mathematical reasoning. At both the high school and college levels female students enroll in fewer advanced and/or optional courses than their male counterparts. Research shows that while there are few sex-related differences toward mathematics per se, significant differences exist in terms of confidence in mathematics ability, stereotyping of professions, parental and teacher expectations, and the perception of the importance of mathematics to future careers and employment.

Minority student enrollment in advanced mathematics courses at the high school and college level indicates that the underrepresentation of minority males (except for Asian Americans) is equal to that of minority females. Some studies show minority enrollment in general mathematics courses to be twice that of enrollment in advanced courses. Minority enrollment and achievement in mathematics is strongly influenced by the fact that these groups tend to be concentrated in urban inner-city schools where the quality of education and resources are poor.

The student problems of lack of motivation, career awareness, and confidence in mathematical ability relate to the fact that recruitment of women and minorities into advanced mathematics (and science) courses and careers has never been a counseling or school priority. Rarely have systematic efforts been conducted to identify, *retain*, and *advance* these groups. Generally, funds have not been targeted for this purpose. Existing policy and fiscal priorities (based on socio-cultural expectations) have fostered the advancement in mathematics and science of those "traditional" students who were either white, male, "gifted", or college bound. At one time the continuing occupational segregation by sex and race was thought to be the result of sexual and racial stereotyping of occupations. As a result of recent studies, it is now attributed, in large measure, to the fact that fewer females and fewer blacks, Hispanics, and American Indians enroll in advanced high school courses by age sixteen. Thus the selection of college

majors and job preparation has been diminished largely by early choices and exclusion or tracking away from mathematics and science.

The perpetuation of school practices and policies discussed above has resulted in few professional role models in science or technological fields who could represent mathematics and science as viable options to minorities and women.

Shortage of Qualified Teachers

While there is an ample supply of teachers for elementary and most secondary school subjects, qualified minority mathematics (and science) teachers are in short supply. In addition, teacher colleges continue to produce "generalists," thereby contributing to the existing pool of teachers with limited training in mathematics and science. In times of budget cuts and declining enrollments, these teachers cannot be assigned to teach in the more specialized areas. In addition to the reforms in teacher education requirements and curricula discussed earlier, new incentives are needed to attract prospective minority teachers to mathematics and the mathematics-based fields.

Corporate and industrial recruitment of high school mathematics teachers represents a major problem area. The lack of cooperation and interchange between the school and corporate sectors has prevented the exploration of joint efforts which might address the problems of teacher training, school curriculum, and industry manpower needs more effectively. A positive example is the IBM-sponsored school assistance program through which IBM technicians and managerial staff are assigned to schools and colleges. This kind of program model could be expanded to an industry-school exchange program which would provide new mathematical and technical expertise to school counselors and students and provide teachers with more advanced mathematical/technical knowledge.

The significant numbers of minority women attempting reentry into the labor market and retraining in new professions represent a pool for recruitment. Programs such as the Women's Reentry Consortium (Fund for the Improvement of Postsecondary Education and National Science Foundation grants), which maintains three regional centers, serve as a hub and resource center for women's reentry programs around the country. They assist in the development of programs and industry placement, and provide seminars and workshops. The programs emphasize scientific, technological, and engineering fields.

Funding and Resource Allocation

In times of fiscal stringency, school administrators generally do not place a priority on advanced mathematics and science courses. Frequently, these are the first to be cancelled.

In addition, most schools must place resources in minimum competency testing efforts and have few remaining funds for expansion of mathematics programs.

Generally state and local funds, which support school districts, are earmarked for particular kinds of students, for example, "disadvantaged," "gifted," and so on. These allocations limit an administration's flexibility to allocate funds to support comprehensive mathematics programs which might improve teacher competency and instruction to students. In some instances Chapter 1 (formerly Title I) tax-levied and special state-funded mathematics programs co-exist in one school; often the efforts fragment or duplicate one another. School and district administrations need the freedom and flexibility to utilize or reallocate funds to develop, expand, and support comprehensive mathematics programs. Several school districts and states (e.g., Oakland, California) have developed fiscal policy and strategies which facilitate program planning and implementation without sacrificing the integrity or the intent of targeted funds.

Finally, school funding policies have not provided for in-service training, incentives for professional development, or released time in any systematic way. Staff training and develop-

ment represent the most immediate solution to increasing teaching competency in mathematics. The lack of funds earmarked for this purpose represents a major problem.

Industry/Employers

As stated earlier, industry has been blamed for recruiting high school mathematics teachers out of the classroom. Yet, they have not reciprocated with any substantial contribution of technical skills and expertise to elementary and secondary schools. Various minority mathematical programs around the country such as MESA (California), PRIME (Philadelphia), and ME3 (New York) provide models for broad corporate fiscal and technical support through joint school and industry efforts. However, these are all too few and corporate support is being withdrawn from many programs at the same time that federal support is being terminated. The previous discussion of the shortage of teachers discusses ways in which industry and the schools can communicate and cooperate to address the problems of mathematics/science education and urban education in general.

The problem of employer bias and discrimination in the hiring of women and minorities is well documented. Recently, in the face of a shrinking pool of mathematicians, scientists and technicians, industry has begun recruitment of qualified males, foreign students, and alien residents. The resulting impact of these practices is that even qualified women and minorities are edged out of the marketplace. This pattern is expected to continue as current enrollment trends show an increase in foreign student graduate enrollment in mathematics, science, and engineering, and a decrease in graduate enrollment for women (mathematics) and minorities (especially blacks) in all these fields.

Parental/Community/Public Issues

The lack of minority parental awareness of the importance of mathematics generally is a result of socioeconomic background, cultural bias, apathy, or misinformation. Since various studies have shown that mathematics enrollment and career choices are related to parental attitudes and level of education, it is important that parents be informed and guided as to the relevance of mathematics and its relationship to future employment and career opportunities. In some instances, parents are aware and concerned, but misguided. A survey of a group of minority parents revealed that they were conscientious in their review of their children's programs to insure that they included some mathematics each semester. The difficulty stemmed from the fact that mathematics courses appeared with numbers (e.g., Math 103) instead of descriptive labels (Algebra) and unsophisticated parents were not aware of the differences between basic mathematics and more advanced courses.

At the University of California, Berkeley, Hispanic students failed to utilize effectively a mathematics retention program that had considerable success with other minorities and women. Further study revealed the presence of cultural bias and parental attitudes which discouraged Hispanic students from actual participation in the mathematics retention program. School staff indicate that these attitudes appear at the secondary level, also. An example of an effective approach to such issues is the Professional Development Program at the University of California, Berkeley which has addressed the foregoing problems through programmatic outreach and involvement of both black and Hispanic parents.

The National Council of Teachers, in its survey of parents, determined that there is considerable parental and lay concern about students' low mathematics achievement. However, it is clear that a range of approaches and efforts will be necessary to address the parental influences effectively.

The lack of public literacy in mathematics is a widespread problem. Already, laymen are struggling to adjust to an increasingly mechanical, digit-coded style of life which extends from

water faucets to banking and money management. In the near future it will be necessary to provide adults with systematic information and training to enable them to keep pace with technological developments and to prepare them to guide the education of their children more effectively.

Recruitment and Retention of Women and Minorities in Mathematics Courses

Currently there is a 15 percent decline in the number of high school graduates annually. Within this decreasing pool there is an even smaller percentage of graduates with three years of college mathematics. All of these students will not continue on to college, as industry is recruiting from this group for specialized training. The recruitment and retention of women and minority students represents a logical means of expanding this limited pool of high school graduates who have completed the advanced mathematics sequence in high school. The most effective target point for pool expansion is the seventh or eighth grade. Intervention at this point allows for the kind of early identification, guidance, career awareness, and supplementary instruction that is necessary to overcome poor attitudes and academic deficiencies. This target point is of particular importance to women. Studies by Kreinberg, Pulos, and State (1976) indicate that carefully designed strategies and activities not only influence adolescent attitudes toward mathematics positively but reverse the pattern of unfavorable attitudes that parallel the adolescent years.

Minority undergraduate college enrollment for the years 1972 through 1979 reveals some interesting and alarming trends in student retention. A Sloan Foundation report by Thomas E. Ford (1980) documents women and minority engineering enrollments and degrees awarded four years later. While the actual number of minority freshmen entering bachelor of science engineering programs increased from 3,000 in 1973 to 9,800 in 1979, the percentage of minority freshmen receiving degrees four to five years later increased only slightly from 3 to 4 percent. Even more alarming is the fact that the percentage of minority students retained during the same period declined steadily from 69 percent in 1972 to 39 percent in 1977. In marked contrast, engineering retention figures for the total population for the same period remained constant at 70 to 72 percent. The Sloan Foundation report suggests that poor preparation, particularly in mathematics, is a major cause of high attrition for minorities pursuing engineering and other sciences. The report also cites the deterioration of inner city schools, where most disadvantaged minorities are located, as a significant factor.

Analysis of the above data in comparison with recent minority enrollment trends suggests an additional factor which may be operating. Since black student enrollment represents a majority of the minority students cited in the study (55-66 percent), it is likely that their college enrollment patterns have a significant impact on the total figures. According to a recent report from the Atlanta University Resource Center for Science and Engineering, the majority of all black students enrolled in colleges were in predominantly white institutions. It is probable that the decline in engineering student retention is related to positive variables in the educational experience at predominantly black colleges which are absent in predominantly white institutions. These successful educational variables (and strategies) should be perpetuated in urban schools to increase retention of women and minority students.

Current Research and Implications for Further Study

Much of the current research on minorities and mathematics assumes that factors effecting differences regarding mathematics learning do not vary within racial and among ethnic groups. In a study of factors which influence enrollment in high school mathematics courses, Matthews (1980) found that sex-related differences did appear within racial groups—"differences which had been obscured previously when analysis was conducted with pooled racial groups." For example, the factor of perceived utility had a strong influence on the decision

to take additional mathematics. Black female students appeared to differentiate between perceived utility for educational goals (college) and perceived utility for future occupation (career or employment). Matthews points out that this distinction may have a negative influence on their decision to enroll in advanced or optional mathematics courses, since they tend to limit their enrollment to those courses required for college. Black males tended not to limit their enrollment to those courses required for college. The study also found sex-related differences among racial and ethnic groups regarding teacher and parental influence on mathematics enrollment. The need for specific data on variation within minority or ethnic groups is underscored also by Vetter (1978). She states that "We know nothing about the makeup of various Hispanic groups' performance in mathematics and science." The current data do not provide breakouts by sex and field within the population. While a higher proportion of Hispanics of Cuban and South American origin have completed four or more years of college, we do not know whether similar ratios exist in the mathematics/science group.

Asian American educators question the broad assumption that they are overrepresented in science and engineering. The "Asian American" population is an extremely diverse one and includes as major groups Chinese, Filipinos, and Japanese and a smaller group of Asian Indians, Koreans, Vietnamese, Hawaiians, Samoans, Guamanians, Cambodians, Micronesians, and Tongans. Susuki in a paper presented at the Congressional Black Caucus Seminar on Women and Minorities in Science and Technology (1980) points out that when figures are adjusted for the number of weeks worked, age, cost of living, and average income, Asian and Pacific Americans earn considerably less than their counterparts among minority groups. Also, there appears to be a difference between the more affluent aliens who constitute 85 to 90 percent of their population's representation in science and engineering, and the remaining 10 to 15 percent who are American born. This latter group, along with some immigrants who enter the USA as young children, has experienced the cumulative effects of historical discrimination and therefore has educational problems similar to those of minorities and disadvantaged women.

Matthews, in recognition of the above variables, cautions against pooling across racial and gender groups that may mask important differences. She recommends also that future research on women and minorities in mathematics include both males and females in the samples.

In line with these recommendations, states or school districts may want to collect and report sex, racial, and ethnic data relating to course enrollment, achievement, attitudes, and career decisions. Such data will assist schools and project directors in identifying problem areas, and in assessing program and project effectiveness.

As Penick (1983) points out, much of the thrust to achieve equitable representation of women and minorities coincided with a decline in the nation's post-Sputnik emphasis on mathematics and science. He suggests that as a result of this void, much of the innovative and exemplary technical instruction in the seventies was, as a result of various foundation and corporate initiatives, focused on two target groups: minorities and women. Extensive data on the design methodology and outcomes of the numerous mathematics and science programs for minorities and women during the past fifteen years has created a recognition of the validity, effectiveness, and transferability of these programs and curricula strategies for all students. In effect, we have learned that much of what enhances mathematics learning for minorities enhances learning for all students.

The National Science Board's (NSB) Commission on Precollege Education in Mathematics, Science, and Technology echoes this belief in its status-of-the-art report. Its inclusion of a special focus on minority and women's programs would seem to suggest that these programs present a valuable resource in addressing the question of how to improve mathematics/science education for all students.

Summary and Conclusions

Two of the problem areas discussed, school-related policy issues and teacher-instructional issues are priority problems in minority mathematics education today. They overlap and interact in such a way that one cannot be addressed in isolation. For example, the lack of qualified mathematics teachers cannot be addressed on any large scale without altering existing practices in teacher education or in-service training. Each of these efforts is a school or institutional policy issue. Negative teacher (or counselor) attitudes can be overcome by mandated school administrative policy or practices such as staff development and course requirements or course enrollment targets.

The majority of issues discussed under the foregoing problem areas stem from existing systems, policies, and practices. While funding can support existing or alternative programs, any broad and lasting change is dependent on the awareness and initiative of those agencies, institutions, and policymakers involved. In order to address any one of the areas it is necessary to determine who is responsible (authority) and who can make the decision and provide the resources for changes (implementation).

Each of the issues of teacher preparation and training, and curriculum, instruction, graduation and competency requirements must be addressed in order to improve mathematics education and learning for minorities. The shortage of qualified teachers, an equally important problem, can be improved through the policy reforms in the areas listed above. Student-centered issues are inherent in each of the priority problems. While the importance of student interventions is recognized, it is clear that we must not hold students responsible for compensating for entrenched policies and practices over which they have no control. The issue of parental and public awareness is integral and therefore supportive of the priority problems; nevertheless, they too can be addressed through policy change, dissemination of information, and outreach.

The area of school and public policy is common to all issues and therefore the most pivotal of those identified. Accordingly, the recommended target audience for future initiatives in mathematics education policy consists of colleges, state departments of education, school superintendents, boards of education, principals and vice principals, and professional teacher organizations.

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MATHEMATICS AND SCIENCE REFORMS: THE ROLE OF THE STATE

Julia Lara

Within the last year, much attention has been focused on the issue of the general decline of academic achievement in the nation's schools. Of particular concern to educators, business leaders, and government officials is the inadequate mathematical and scientific knowledge base of today's high school graduates. Since 1973 mathematics and science participation rates at the high school level have been on the decline. This decline has been accompanied by a decrease in the number of advanced mathematics and science courses offered by schools and the decrease in the number of qualified mathematics and science teachers at the secondary level. These facts have been dramatized in the recent national reports on the state of education in the nation, including the report of the National Commission on Excellence in Education. These reports agree that, given the impact that scientific and technological advances are having on production and in service enterprises, it is critical that the nation produce a citizenry with an understanding of scientific concepts and technological knowledge—knowledge which is functional at different levels of the occupational sector, including the least specialized levels.

A flurry of activity at the state and local levels has resulted, particularly in response to the report of the National Commission on Excellence in Education, which calls attention to the need to strengthen American public education. School systems across the nation are being challenged to provide training in advanced mathematics and science courses to a large number of students in order to insure that there are enough graduates capable of filling the increasing number of jobs demanding these skills (*Making the grade* 1983, 14).

This paper presents an overview of state-sponsored initiatives to improve mathematics and science instruction which were being considered for implementation in late 1983 and early 1984. Specifically, it refers to those recommendations which were the outcome of the many task forces and commissions authorized by governors and state legislatures. These recommendations will not necessarily become law until they are enacted by the state boards of education and/or the state legislatures. Also, and where appropriate, a discussion of the probable impact that these proposed efforts might have on minority students will be discussed briefly.

State Role

The U.S. Constitution does not contain a provision which delegates authority for education to the federal government. Consequently, public education has been considered a state function. The state legislatures, with authority from the state constitutions, have established a decentralized system of educational governance in which responsibility for carrying out the goals of American education is shared by the state and local governments. The local school districts have been delegated the responsibility for curriculum planning and development. State education agencies, as the administrative arms of the legislatures, are responsible for the overall supervision of public elementary and secondary education throughout each state.

In addition to their regulatory function, the state education agencies serve as critical links between local school districts and institutions of higher education, and are the principal force shaping each state's teacher pre-service and in-service training programs. To insure that minimum educational standards are met by the local school districts, the states also establish minimum program requirements, teacher certification requirements, and curriculum guides.

Student-Centered Initiatives

According to a survey conducted by the Education Commission of the States, 275 state-level task forces had been convened in fifty states by 1983 (*The nation responds* 1984, 15). These task forces have recommended a broad range of educational improvement programs to the legislatures. If approved, these initiatives will be implemented by the local school districts. The following is a brief summary of a number of proposed initiatives which impact mathematics and science learning and teaching.

Recommendations of greatest appeal to educators and the legislature are those focused on curriculum reform and graduation requirements. Thirty-five states have enacted legislation to increase the mathematics and science units required for graduation.

Curricular reforms range from the redefinition of basic skills to include mathematics and science in grades kindergarten through eight to the development of courses of study for all subject areas (kindergarten through twelve). The recommended state curricular guides or syllabi in mathematics have been modified to focus on more complex mathematics skills: reasoning skills and the logical process and application of mathematical reasoning skills to other disciplines. In science more emphasis is given to the hands-on approach to learning with opportunity for experimentation and individual problem solving (*New York State Board of Regents proposed plan 1984).*

The State of Florida, among others, has enacted a multi-pronged approach to the mathematics and science education problem. Student-centered initiatives consist of curriculum revision to provide students with greater exposure to laboratory experiences in mathematics, science, and computer education in grades kindergarten through twelve, an increase in the mathematics/science graduation requirements to three units, summer camps in science, mathematics, and computer literacy, and out-of-school academic enrichment programs.

The use of testing as the primary standard for academic success has once again generated a great deal of controversy. Although competency testing had been established in forty states prior to the "excellence" thrust, more recent proposals intend to broaden the scope of these examinations (U.S. Department of Education, *The condition of education* 1984, 179). At least twenty-two states are contemplating the practice of using performance on norm-referenced competency tests as a criterion for high school graduation. Under this policy, students who fail to meet minimum passing scores will be denied a high school diploma and will instead receive a certificate of attendance.

Critics of this policy argue that the use of these tests will have an adverse effect on minority students. Research conducted on the impact of competency-based testing has shown that far more minority students fail to meet the minimum passing score than majority students. In the State of Florida, which has been involved in the minimum competency test controversy since 1977, 57 percent of the 1983 senior class who failed the test were black students. Black seniors comprised 20 percent of the class (*Education Daily* 1984). When used as a requirement for graduation, the effect on minority students will be punitive and may encourage student dropout.

The attrition rate among minority students is a concern in districts with a significant percentage of minority students. Hispanic and Native American students have the highest dropout rates at 40 percent and up to 50 percent, respectively. Recent figures for black students indicate that the dropout rate has risen to 24 percent from a 1976 level of 20 percent (Steinberg, Lin Blinde, and Chan 1984, 115). Even if students were to be granted a certificate of attendance, the educational and employment opportunities available to them would be limited. They would be denied access to institutions of higher education. If such students were later to decide to pursue college level work, their efforts to do so would prove even more difficult since colleges and universities around the nation are now setting more demanding admissions requirements. (As of February 1985, seventeen states had, in fact, raised their college admissions requirements (*Changing course* 1985, 11).) A non-

punitive approach to the assessment of academic progress issue is to use competency tests as diagnostic and prescriptive tools rather than for classification purposes.

The specialized high school models (e.g., North Carolina High School of Science and Mathematics, Bronx High School of Science) have been replicated in a number of states (California, Georgia, New Mexico, Ohio, South Carolina, Tennessee, Virginia). These schools generally have a demanding curriculum and have been designed for "gifted" high school students with special talents in mathematics and science. Entrance requirements include a battery of aptitude tests which ensure that only the academically gifted are admitted. Although highly successful in attracting the top students of the college-bound population, it is doubtful that these schools will significantly increase the pool of students available for mathematics-based careers given their entrance requirements. On the other hand, mathematics/science magnet schools are more likely to provide high quality education to average ability students who also want to specialize in a mathematics-based career. Recent studies have indicated that students in magnet schools have a lower absenteeism rate and score higher on achievement tests than their peers in regular classes at every grade level (Gratta 1983). Also, a study funded by the Department of Education on magnet schools concluded that "the high quality education was not related to selectivity in student admissions" (Lowry 1983).

Excellence in various subject areas (particularly in mathematics and science) will be encouraged and recognized in about one-third of the states by the development of out-of-school academic enrichment programs, summer residential institutes in mathematics and science, and statewide academic competition programs. (Twenty-four states have enacted or proposed these projects.) The criteria for the awards and for the selection of students vary, but generally students must demonstrate a pattern of outstanding performance in the subject area: high grade point average, high test scores, and a certain number of advanced courses taken. If the President's Academic Fitness Awards are indicative of local district response, this approach is likely to stimulate a great deal of interest among the local school districts, which are ultimately responsible for recommending the students.

Programs that reward and recognize growth or progress will be more successful in encouraging academic excellence among students at all levels of ability than those targeted at the gifted and talented.

Teacher-Centered Initiatives

As previously noted, the states are in a position to influence the teaching profession in a number of ways. States set requirements for teacher preparation and certification, thereby setting the standards which teachers must meet to enter and remain in the profession. They also play a role in improving teaching by providing additional training in high demand areas such as mathematics and science.

In response to the call for reforms to improve the quality of teaching, the states have proposed a number of remedies. Across the board salary increases, merit pay, changes in the certification procedures, and an increase in the teacher development activities are among the measures that have been introduced to the legislatures.

Two state-sponsored, teacher-centered programs have come under attack: differential pay (merit pay) and the use of competency testing for teacher certification. Under the merit pay plan, teacher salaries will depend on the proficiency of the teacher in contributing to the education of his/her students (Institute for Research on Education, Finance and Governance, 10). The plan is designed to reward exemplary teachers and to retain those teachers in shortage areas (mathematics/science).

Merit pay is not viewed as an effective method of improving instruction or of retaining good teachers. Studies have not found a relationship between higher salaries and student achievement. Additionally, it has been very difficult to design an objective mechanism

for evaluating teacher performance which would serve as the criterion for differential pay. The social status of the teaching profession and the working conditions of teachers are considered, by some teachers, to be just as important as salary increases.

Since 1982, thirty states have initiated a plan for teacher certification which requires some form of competency testing. Regardless of the test used (National Teachers Examination or other standardized instrument), this policy will meet with opposition from some minority educators. Competency testing for certification may decrease the number of black candidates for the teaching profession as well as the number of those applying for recertification. Ironically, teaching has been one of the few professions open to blacks, particularly in the South.

A recent study of the Louisiana testing program concluded that since the beginning of the competency testing program in 1978, of the 1,394 black students from public institutions who took the National Teachers Examination, only 15 percent achieved a passing score. The success rate for black students at black institutions is even lower (10 percent). It is clear that this policy is having a disproportionately negative impact on black aspirants to the teaching profession. Unless the states design a mechanism of awarding teacher certificates which does not rely primarily on standardized test results, black students will be denied role models, a factor which studies have found to be critical in the development of a positive self concept.

Several programs have been developed at the state level to address the mathematics/science teacher shortage crisis. To encourage mathematics/science majors to select teaching as a career, the states have proposed scholarships or loan forgiveness programs whereby a student's college tuition is paid in full or in part with an assurance that he/she will teach mathematics and science for a certain number of years in the public school system.

The private sector has collaborated with state agencies and universities by permitting their scientists either to teach a mathematics/science course or to train teachers in need of further mathematics/science skills development. Additionally, retired scientists and engineers have been given "provisional" teaching certificates. Some state legislatures have proposed the establishment of summer institutes for teachers in science, mathematics, and computer technology.

The state university systems have also responded to the mathematics/science teaching crisis by developing training programs for out-of-field teachers. Under this program teachers in non-shortage areas are required to take a certain number of courses in mathematics/science (twelve to fifteen credits) in order to obtain certification as mathematics/science teachers.

The states are to be commended for responding to the call for excellence by proposing a number of solutions which address education reform comprehensively: student-centered as well as teacher-centered initiatives. However, educators concerned with excellence *and* equity ought to be cautious when embracing proposed solutions which are targeted to a sector of the student body which is best prepared and therefore will insure the success of these initiatives. Elitist solutions such as systematic tracking by early test scores and specialized mathematics/ science programs for minorities from the teaching profession which use competency tests will not solve the problems of the public school system but will instead reverse the gains made within the last fifteen years.

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ADDRESSING THE CRISIS IN PRECOLLEGIATE MATHEMATICS AND SCIENCE EDUCATION

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Briefing for the House of Representatives' Democratic Caucus. The briefing was organized by the Population Resource Center and held on 27 April 1983. This material was drawn from a forthcoming longer paper by the author, **Scientists Interested in Education and Educators Interested in Science: Bridging the Gap.** *The author thanks Ami Zusman for her assistance in the preparation of this statement.*

The current crisis in the quality and instruction of precollegiate mathematics and science education is now acknowledged to be one of the major problems confronting American education—and, by logical extension, American society in general. Economic, demographic, and social changes have created this crisis. As I will discuss shortly, demographic changes will make this crisis even more acute in the future, unless we take steps to alleviate it. My purpose here is to describe this crisis and to argue that we cannot approach it with the simplistic and short-term "solutions" we sought to apply to the similarly perceived crisis in the post-Sputnik era. Money is not enough. Unless policy initiatives deal with underlying problems and draw on the skills and expertise of the scientific and technological community, we likely will find ourselves in ten to fifteen years where we found ourselves ten to fifteen years after Sputnik—with the problem no closer to resolution.

The Problem

The evidence that a crisis in precollegiate mathematics and science education exists is seemingly incontrovertible:

> A 1981 National Science Teachers Association survey reported that, nationally, over half of newly employed science and mathematics teachers in secondary schools were employed on an emergency basis because no qualified teachers could be found (Guthrie and Zusman 1982, 7).
>
> Mathematics and science teachers are leaving the teaching profession for jobs in industry at a rate four to five times their rate of replacement (Wicker 1983, a-21).
>
> More than 50 percent of the students receiving degrees in 1981, qualifying them to teach mathematics or science in secondary schools, went into industry (Institute for School Development 1983, 9).
>
> Between 1966 and 1980, the percentage of freshman students planning to major in mathematics and physical science fell from 7.8 percent to 2.6 percent (Buccino 1982, 66).
>
> The percentage of college-bound high school seniors scoring 650 and above on the mathematics portion of the Scholastic Aptitude Test (SAT) declined by 23.4 percent, while the proportion scoring fewer than 300 points increased by 38 percent during the ten-year period 1972-82 (*Chronicle of Higher Education* 1983, 7).
>
> Since 1960, enrollment in remedial mathematics courses in universities and four-year colleges has increased by 165 percent. Remedial courses now constitute 36 percent of

all mathematics enrollments in colleges and universities, compared to 13 percent in 1960 (Buccino 1982, 77).

The absolute disparity in scores on the mathematics portion of the SAT for college-bound seniors planning to major in education and seniors planning majors in all disciplines increased by 50 percent between 1973 and 1981. In 1973 the average mathematics SAT score for all majors was 481; for prospective education majors, it was 449. In 1981, the scores were 466 and 418 respectively — a difference of 48 points (Buccino 1982, 125).

Universities are relying increasingly on foreign students to provide new Ph.D.s in engineering, computer science, mathematics, and the physical sciences. According to the National Research Council, in 1981 more than 51.5 percent of the new Ph.D.s in engineering were foreign students. The numbers for computer science, mathematics, and the physical sciences were 25.6, 32.3, and 22.0 percent respectively (*Chronicle of Higher Education* 1982, 8).

Future demographic changes will make this crisis even more acute. Three trends in particular have implications for mathematics and science education. First, school enrollments, which for over a decade have been declining, will begin rising again starting in fall 1983, as the children of the baby boom generation enter school. By the end of this decade, larger numbers of high school students will be ready to take mathematics and science courses (Guthrie and Zusman 1982, 4).

Second, even while school enrollments climb, the traditional pool for student teachers will continue to shrink. This is the result of several factors. The number of persons turning twenty-two years old (the typical age for entering teaching) will begin to decline about 4 percent annually starting in 1986 (Morrison 1976, 20). The reserve pool of teachers we once could count on — mostly women prepared to reenter the labor market after several years' absence— will contract because relatively few college graduates with teacher preparation training are now entering this reserve. And, most important, expanding career opportunities for women and minorities mean that we no longer can — nor should—rely on discriminatory practices to bring a disproportionately high number of women and minorities into the teaching profession. The decline in the overall teaching pool is likely to be exacerbated in mathematics and science fields, for the reasons I have discussed.

Third, immigrant and minority group students, many of whom in the past have taken fewer mathematics and science courses than other students, will comprise an increasing proportion of our school-age population. Unless we develop new approaches to stimulate interest in mathematics and science fields by these students as well as by women, who also have entered mathematics and science fields in disproportionately small numbers, the nation may be unable to achieve its fullest scientific and technological potential.

If the trends of declining student interest in mathematics and science, plunging scores on achievement examinations, and a diminishing supply of qualified precollegiate mathematics and science teachers continue or worsen, the United States shortly may find its ability to maintain its leadership in science and technology irreversibly compromised. The consequences would be disastrous not only for the nation's future economic performance but for its social, political, and intellectual fabric as well. Advanced training in mathematics and science is central to the growth of high-technology industry and businesses, seen by many as the most promising economic sector for the U.S. at a time when the old low-technology industries in steel, automobiles, and the like are experiencing high unemployment rates. Advanced technology is critical to maintaining the increasingly sophisticated apparatus needed for national security. Conversely, inadequate mathematics and science education limits the economic advancement and career opportunities of individuals, a factor which has widened the economic gap between minority and non-minority, female and male; and it lowers the economic productivity of the nation as a

whole. Analysts have argued that the decline in the quality of science and mathematics instruction has already impaired the ability of the citizenry to exercise informed judgment about issues requiring basic scientific literacy, weakened the nation's capacity to prepare its youth for life in a technology-intensive future, and threatened U.S. businesses' competitive edge over countries such as West Germany and Japan, where high importance is placed on mathematics and science education (White 1982, 11).

The Supply-Side Solution: Problem and Prospects

Would-be educational reformers have offered a number of suggestions to deal with the crisis in precollegiate mathematics and science education. One frequent proposal is to make massive public investments in the production of more and better prepared mathematics and science teachers. The most often suggested investments are tuition scholarships for prospective classroom recruits and incentives for colleges and universities to improve their programs in mathematics and science education for prospective teachers. Suggested solutions, not unlike those applied during the post-Sputnik period of reform, also include subsidized loans for prospective teachers and retraining grants for uncertified mathematics and science teachers willing to upgrade their skills.

Though essential, these initiatives will not be sufficient. First, any strategy which focuses solely on prospective classroom teachers and not on the teachers now in the classroom will have no appreciable effect for several years. Second, the shortage of qualified and competent mathematics and science teachers is due to a set of factors which will remain unaffected by supply-side solutions. The most important of these factors are the following:

1. *The abysmally low starting salaries of prospective mathematics and science teachers, compared to salaries they would command in industry.* This problem, which has decreased the attractiveness of the education profession in all subject areas, produces particularly perverse results in the case of qualified potential science and mathematics teachers. The more qualified the prospective teacher, the greater is that teacher's potential for nurturing the talents of the young. The larger the classroom-industry gap, however, the lower the probability that the critically-needed prospective teacher will end up in the classroom. In other words, the well-qualified would-be teacher, who is potentially capable of doing the greatest good for the greatest number, is being asked to make the greatest financial sacrifice. Is it any wonder that not only are the brightest students shunning careers in mathematics and science education, but the most academically able teachers are leaving the profession in disproportionate numbers (Toch 1982, 14)?

2. *The prospect every beginning science or mathematics teacher faces of being cut off from new disciplinary developments and breakthroughs within a few years after entering classroom service.* The struggle the ordinary classroom teacher engages in to make science and mathematics accessible to students leaves little time for advanced study or laboratory work. Moreover, the teacher who prevails against the odds and continues to improve his or her understanding of the subject area will rarely be given an opportunity to use this new knowledge in the classroom, because most science and mathematics teaching is directed at students studying "basic" science and "elementary" mathematics. The sharp decline in the number of high school students taking advanced placement courses in science and mathematics has made this problem even more acute over the last decade.

The teachers who are most likely to be bothered by the specter of intellectual obsolescence and isolation from the world of the practicing mathematician and scientist are also those most likely to abandon the classroom for employment opportunities which permit them to practice science, or to put their scientific and analytical training to more remunerative uses.

3. *The low prestige of precollegiate science and mathematics teachers within the science community.* In spite of the long-existing classroom-industry salary gap, many highly qualified prospective or practicing teachers who have left the school system had earlier been willing to—

and some did—forego income in exchange for the opportunity to serve in the classroom. What, then, altered the early preferences of those who have now forsaken the classroom for industry? There may be several reasons, including the expansion of job opportunities outside education for women and minorities, a growth in technology-oriented business generally, and a real or perceived decline in the working conditions in the schools. However, I believe that a large part of the reason for this classroom exodus is that the psychic benefits and satisfactions which prospective and practicing mathematics and science teachers derive from classroom service often diminish with time.

The problem of psychic depletion is compounded by the relatively low standing of precollegiate teachers of mathematics and science within the larger scientific community and the constant barrage of criticism directed at them by members of the scientific community, policy makers, and others. Except in unusual cases, university-based professors, who think of themselves as "scientists interested in education," rarely come into respectful contact with classroom teachers who, in generous moments, are described as "educators interested in science." When professors do communicate with their classroom counterparts, they usually do so to complain about the sorry state of mathematics and science education in the lower schools.

Faced with a steady cascade of criticism, it is hardly surprising that the classroom teacher turns away from professional organizations dominated by practicing scientists. This only further compounds the feeling of the professoriate that classroom teachers are not serious about science and thus are not to be taken seriously as significant links in the chain of scientific discovery and development.

Let us examine the policy recommendations that have been offered to alleviate the deleterious impact of the salary gap, the isolation of teachers from the practice of science, and low status within the scientific community.

Two suggestions for solving the salary gap problem have been offered: The first is across-the-board increases in teacher salaries, and the second is salary differentials for mathematics and science teachers. Unfortunately, the chance of either of these alternatives being implemented in the near future is slim. Thus, the chances of ever finding out just how important salaries are in influencing the personal incentive calculus of "would-be" and "used-to-be" teachers are just as slim. Given the current national economic climate, across-the-board salary increases for all teachers, sufficiently large to channel prospective mathematics and science teachers into the classroom and to retain the best and brightest already in the classroom, would be prohibitively expensive. Differential salary schedules for mathematics and science teachers, while less costly, have been vigorously opposed by teachers' unions and professional teachers' associations representing other disciplinary interests. And not without just cause. The relationship between supply and significance is not at all obvious.

The problem of intellectual obsolescence, alienation, and isolation is, in theory, easier to solve than the salary-gap problem. Initiatives that would reduce the magnitude of the former mirror those applied during the post-Sputnik period of reform. They include:

Re-establishing the summer mathematics and science institutes financed by the National Science Foundation (NSF) at the level that existed during the mid-sixties. These institutes should be organized and staffed by scientists and mathematicians as well as educators. They should concentrate their efforts on subject matter material, curriculum experimentation, improvement of pedagogical skills, and technology-assisted instruction.

Providing summer employment opportunities for selected mathematics and science teachers in research universities, national laboratories, and research laboratories in private industry. The federal government can directly influence the behavior of universities and national laboratories by utilizing a variety of incentives, including partial salary subsidies for such employment. In the case of private industry, tax credits equal to perhaps 25 percent of actual salary expenditures should produce an ample

supply of employment-training opportunities for qualified classroom teachers.

"Encouraging" school districts to support talented mathematics and science teachers willing to invest extra time and energy repairing or constructing needed equipment, preparing special demonstration exhibits, or acquiring new skills, such as using computers to teach problem-solving skills. This support could take a number of forms, including released time, extra sabbatical credit, mini-grants for special projects, or fellowships to support further study.

These initiatives carry certain risks. Some of these efforts, for example, may only increase the level of dissatisfaction and discomfort by exposing talented and highly competent teachers to the opportunities that would be theirs if only they would abandon the classroom. Others may even raise the potentially divisive issue of unfair preferential treatment of one class of teachers over teachers in less favored subject areas. However, it is difficult to imagine that the consequences of implementing these modest reforms would be worse than the inevitable consequences of doing nothing.

The low status of precollegiate teachers of mathematics and science within the scientific community may be as formidable a problem to resolve as the classroom-industry salary gap. Some steps that could be taken to alleviate this problem include:

Inviting mathematics and science teachers to play a more active role in professional scientific organizations.

Fostering closer relationships between professors in science and mathematics departments and professors in departments of education charged with the task of preparing precollegiate teachers of mathematics and science.

Encouraging professional scientific organizations to play an active role in establishing standards for precollegiate mathematics and science teachers, developing curriculum, and monitoring and evaluating instructional objectives and performance.

Last, but not least, the professoriate and other members of the scientific elite should meet the needs that exist among mathematics and science teachers for recognition and reward by acknowledging the contributions of classroom teachers through well-publicized awards ceremonies for outstanding service and performance.

The real questions to be answered are these: (1) "Will implementation of the supply-side solution (for example, through tuition scholarships or re-training) arrest and reverse the ominous trends in precollegiate mathematics and science education?" (2) "Will reforms intended to deal with the problems of low starting salaries, intellectual isolation, psychic depletion, and low prestige work?" I am not convinced that they will. Supply-side policy initiatives are unlikely to be sufficient, given the factors inhibiting teacher supply. And reforms extensive enough to overcome these barriers may be too costly to implement or may require fundamental alterations in existing relationships unlikely to be obtained (i.e., reordering the relationship between precollegiate teachers and university professors, prestige sharing).

Recommendations for Policy Initiatives

We cannot "solve" the crisis in mathematics and science education. However, we may be able to provide some solutions to smaller problems, aspects of this crisis that are under our control. Moreover, even these problems will not be solved without the significant involvement of the university scientific community. Nevertheless, permit me to offer a few potentially resolvable problems and those solutions to which the federal government can contribute. I will

first describe a solution which, while radical, should have an immediate and substantial impact on the shortage of qualified mathematics and science instructors in the classroom. Because its impact would be felt almost immediately—not more than a year after steps to implement it were first taken—I will describe this solution in some detail. Then I will sketch more briefly other solutions to aspects of the crisis which should also be implemented—solutions likely to have a moderate impact in the mid-term or those which could eventually have substantial impact but which are long-range solutions.

Problem: *To increase the flow of highly qualified and enthusiastic mathematics and science teachers into the nation's classrooms.*

Solution: *Establish incentives to encourage and support advanced, pedagogically qualified graduate students in mathematics, science, and technology to take on part-time teaching assignments, especially at the junior high and secondary school level. The federal government can contribute to such programs by providing supplemental funding and legitimacy for them.*

The average junior high and secondary school teacher of mathematics or science instructs students for 25 forty-five-minute periods per week. A full-time-equivalent teacher can quite easily be obtained by combining the services of three graduate students, whom we will call Graduate Student Instructors (GSIs), each of whom would teach an average of slightly more than 8 forty-five-minute periods per week.

School districts would pay most of the costs for GSIs by utilizing funds currently being used to pay the salaries of unqualified teachers of mathematics and science. However, school districts should not shoulder the entire financial costs associated with the employment of Graduate Student Instructors.

The federal government could assist the development and expansion of GSI programs by awarding supplementary funding to the GSI's department for performing an extraordinary public service. Not only would such funding increase the likelihood that graduate departments would participate in such programs, it would also increase the legitimacy of joint school-university responsibility for precollegiate mathematics and science education and would provide greater visibility for the GSI concept. The National Science Foundation unit charged with designing programs to improve the quality of precollegiate mathematics and science instruction would be a logical agency to administer such federal funds.

These federal funds might be provided on condition that the university graduate departments, which would benefit from the GSI program by obtaining financial support as well as training for their students, pay some share of the GSI's stipend. Requiring graduate departments to pay part of the GSI's stipend would help guarantee that the potential GSI's graduate advisers and graduate departments will treat precollegiate teaching as a "legitimate" graduate student activity. Graduate advisers and departments would have to exercise some judgment in allocating their resources, an exercise which is bound to produce better decision-making. Where departments now require their graduate students to serve a period of time as teaching assistants in the university, teaching mathematics and science in the high school should be a fully accepted alternative.

The impact of the GSI concept on helping to alleviate the mathematics and science teacher shortage will not be trivial. For example, in 1982 the University of California, Berkeley had only five individuals in training as science teachers and only one student preparing to be a secondary school mathematics teacher (Guthrie and Zusman 1982, 1). If less than one percent of Berkeley's roughly 4,000 graduate students in mathematics, science, and technology were to take part in the GSI program, the increase in Berkeley's contributions to precollegiate mathematics and science education would be marked.

Problem: *To keep mathematics and science instruction and instructors current with changing developments in the field.*

Solution (1): *Provide federal funding to university schools of education to enable them to employ scientific and technical personnel from private industry on a part-time or short-term basis. Such industry personnel would instruct teacher-credential candidates in the areas of their specialization and*

would collaborate with education professors and researchers on the development of new instructional devices and software.

The rapid developments occurring in science and technology could be made more accessible to precollegiate teachers if teachers and prospective teachers were brought into close contact with those working in these changing fields. The expertise of industry workers is a resource too little tapped below the college level. Some have suggested employing industry employees in high schools themselves—a suggestion with merit. However, by bringing industry experts into teacher-training programs, schools of education could produce a multiplier effect, spreading current information and new technology well beyond the confines of a single high school classroom. The amounts of federal funds needed for this effort would be relatively small; some of the costs should also be borne by industry, which should be persuaded that it is in its long-term interest to provide employee release time or other assistance.

Schools of education could also bring together technical experts from industries with the potential to have a large impact on instruction—especially from the computer industry as well as from manufacturers of video disks and the like—and university educators and researchers. Too often those with expertise in student learning processes and those familiar with the developing technological tools which could be applied to aid student learning have little interaction. Currently, for example, computer-aided instruction programs rarely go beyond routine drills and introduction to a subject matter. A second problem is money: The costs of designing and developing technologically-based instructional programs or devices are likely to be beyond the budgets of most schools of education. Through collaboration between technical experts from industry and educational researchers in universities, it is likely that far more sophisticated use of the new technology, geared toward real educational needs, could be developed.

Solution (2): *Support other programs and means (noted earlier) to reduce current teachers' intellectual isolation. These include National Science Foundation-sponsored summer mathematics and science institutes, tax credits to industry and subsidies to universities for providing summer jobs to teachers, and funds to school districts to increase incentives for teachers to upgrade their skills or develop new curricula.*

As discussed earlier, these steps carry risks; and, by themselves, they are insufficient to overcome the problem of teacher isolation and obsolescence. Nevertheless, if they can be linked to a significant willingness by the scientific community to interact seriously with classroom teachers, they may help alleviate the problem.

Problem: *To reduce the shortage of qualified scientific and technical workers at all levels, while ameliorating the effects of technological displacement.*

Solution: *Establish an Educational Training Fund, along the lines of the Unemployment Compensation Fund, targeted to employees in technology-sensitive industries.*

This final recommendation is radical, long-term, and comprehensive. It goes beyond the previously described concerns for precollegiate mathematics and science education. It will not be easy to implement. Nevertheless, it has a large potential for helping shift workers from areas with declining production to those with growing demand. Under this proposal, employers in industries sensitive to technological change would contribute money equal to a small percentage of an employee's salary to a special fund to be used only for training. The longer an employee worked, the more money would accumulate in the special fund. Employees would be eligible to use the money in the fund to offset tuition or related costs for new educational training. The fund would benefit employers as well as employees by encouraging employees to keep abreast of technological developments. Although programs would have to be certified as eligible, training should be broadly defined, and individuals should be able to choose both in what area the training would be and where it would be given.

This proposal raises a number of questions which would have to be answered. How can we make training programs more attractive to adults than other somewhat related efforts have been, such as those in Europe to give working adults preference in university admissions? Should individuals in certified training programs also receive unemployment until after their

training is completed? How does one improve the chances that the new training will meet employment needs, without narrowly constraining the individual's choices?

I hope this examination of the condition of precollegiate mathematics and science education, its implications for the rest of our society and economy, and some solutions to some aspects of the crisis will open up further discussion of the issues.

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TOWARD A MATHEMATICS CURRICULUM IN THE FORM OF THE CORE AND OPTION-MODULES

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This paper was originally read at the International Conference on Mathematics Education (ICME 5) in Adelaide, Australia, 1984.

Objectives and Outlines

In a separate paper by two of the authors (Fujita and Terada, manuscript), the necessity of and background to organizing a flexible curriculum for high school mathematics in Japan has been described in detail. In this paper we will merely summarize the main relevant points.

First, we have adopted as the main goal of mathematics education in Japanese high schools the cultivation of mathematical intelligence. In organizing a mathematics curriculum, which becomes a national one when adopted by the Ministry of Education, we must honestly take into account the existence of two groups of high school students. The first group, the majority in number, is composed of those students who must acquire so-called mathematics literacy, that is, a standard competency in mathematics for citizens to live and work in a society with modern technology. The second group is composed of students of letter aptitude (moderately elite) who are expected to develop their potential in mathematics to the utmost so that they can successfully proceed to university study and future professions in various fields of natural and social science and technology.

In Japan it is not considered appropriate to separate these two groups in any apparent way at the initial stage of high school education for two reasons. The psychology of the students and of Japanese society does not favor an explicit differentiation according to aptitude, and we find that in the first grade of high school many slow starters can catch up with and surpass their peers if they are properly guided. Moreover, average students will decide the direction of their future careers a little later. Thus the formal structure of our intended curriculum should be a "one track" main line supplemented by side branches and extensions which can make the whole curriculum rich and flexible. These considerations influenced our organization of the "core and option-modules" curriculum (hereafter abbreviated as COM curriculum).

The main line of the COM curriculum is called the core. It should be learned by all students, and, as mentioned above, will provide the majority of students with a modern and sound mathematics literacy as a basic part of mathematical intelligence. On the other hand, the side branches and extensions correspond to optional modules in the COM curriculum. Brighter students can study some of the optional models, which are chosen in conformity with their abilities and aspirations, while they are learning the core. This will prevent the brighter students from being limited by materials too easy for them. These optional modules might be called the side options and those advanced topics which should be learned by the brighter students or students bound for science and technology after completion of the study of the core might be called the advanced options. Obviously, depending on the level of the school or the class, one and the same optional module can be used as a side option or as an advanced option. In some cases, we may have to prepare remedial topics as optional modules. Symbolically, the structure of our COM curriculum is illustrated below.

Needless to say, the most crucial point on which the success of this kind of curriculum depends is the contents of the core and option modules. As a matter of fact, our study group on mathematical education is still discussing these matters and investigating the merits and defects of various proposals. However, there are a number of items which have either been adopted or agreed upon as targets of a further study. These will be described in the following sections.

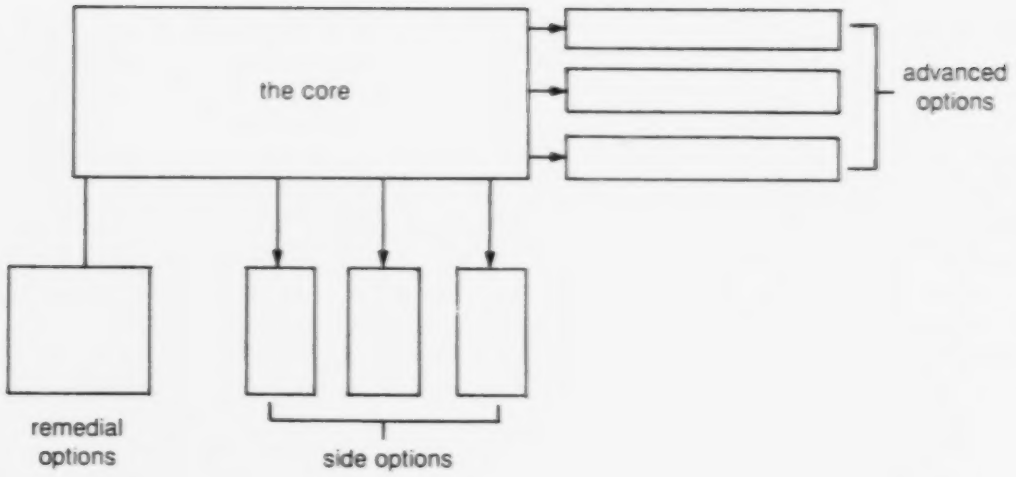

Figure 1

Let us mention here that if the core is successfully organized, it can also be regarded as a standard of mathematical culture and as a background on which retraining or adult education in mathematics can be based.

Finally, once the COM curriculum is adopted, then the role or responsibility of each school in organizing its school curriculum must be emphasized. According to existing curricula, the contents and level of school instruction of subjects are in principle left to each school within the framework of the national curriculum, the "Course of Study." However, there is very little room at each school's disposal regarding the selection and order of topics to teach. Thus the COM curriculum will encourage school teachers to devise school curriculum best fitted to their schools, although initially this unfamiliar freedom may cause confusion in some local schools. In addition, the flexibility of the COM curriculum, and in particular the extensive but selective study of the option-modules, will evoke considerable dispute over deciding the range of topics from which problems for university entrance examinations are created. In light of the educational environments in Japan these are not trivial problems. However, we must resolve them in some way or other, for we believe that a mathematics curriculum of the COM type is a necessity in the coming years.

Strategies for Organizing the Core

Our study group is now developing a syllabus for the core of the COM curriculum, taking note of possible topics to be included in the option-modules. Obviously, the whole COM curriculum should be completed as an integrated system. However, as an interim report, we describe in this section our strategies for organizing the core, and in the following section we describe its contents with remarks on the points to be discussed further.

(1) The core is composed of topics which should be learned by all students during their high school attendance. The knowledge and the ability in mathematics to be acquired by learning the core is at the level of standard competency for an intelligent citizen in a society with modern technology. The majority of students, perhaps 70 percent of the age cohort, should master the core topics.

(2) The topics in the core should be organized to form a compulsory subject in a consistent and integrated form.

(3) According to the current curriculum in Japan, the compulsory subject of mathematics includes basic topics as well as some difficult topics which are only provisions to be used at much later stages. We should exclude from the core of the COM curriculum those difficult provisions which are not needed in the initial lessons and which often serve as obstacles or hur-

dles that hinder average students' study at the half-way point or that make them lose their confidence in mathematics. Most of the technical provisions which used to be included among the compulsory subjects can be taught with better motivation when and where they are needed.

(4) In order to assist low attainers or slow starters in understanding mathematical notions, graphical and/or numerical approaches should be recommended. An active use of pocket calculators should be introduced to this end. For instance, without any rigorous argument, all students can recognize the notion of the limit and iterative generation of sequences by means of numerical experiments using calculators. For example,

$$a_{n+1}=\sqrt{a_n},\quad a_{n+1}=\frac{1}{2}a_n,\quad a_{n+1}=\frac{1}{2}\left(a_n+\frac{2}{a_n}\right).$$

(5) The core as well as option-modules should be enriched by inspiring and/or enjoyable problems (examples and exercises). These should aid students' self-learning. We agree that problem-solving is one of the most efficient means to cultivate mathematical intelligence at any level.

(6) With the rapid progress of information technologies and the resulting spread of the new media, the style of textbooks and that of teaching-learning may change remarkably. These factors should facilitate implementation of our flexible COM curriculum by offering ample opportunities for self-learning, personalized learning, or small-group learning. The establishment of national or local centers which compile, collect, store, and disseminate teaching programs, varieties of problems, and other data is also recommended. School teachers should be encouraged to participate in these activities.

A Tentative Plan of the Core

We now present our tentative and incomplete plan of the core. The topics in this core part have been selected essentially from the topics covered by current subjects (compulsory or optional) according to the present course of study, the national curriculum. In this sense, this initial plan is rather conservative and we feel that if the next reform of the national curriculum does not take place along the line of our COM in the near future, then the core part will be innovated considerably. (See Appendix A of the paper by Fujita and Terada (manuscript) concerning the contents of the present national curriculum of senior high schools in Japan.)

As to the background in mathematics of students at the time of enrollment in senior high schools, for the time being we assume the contents set by the course of study for junior high schools, of which some detailed information is given in Appendix B of the Fujita and Terada paper (manuscript).

In our tentative plan, the core consists of the following six topics. The title of each topic is merely provisional.

C-1 Quadratic Functions
C-2 Finite Mathematics
C-3 Measurement of Geometric Figures
C-4 Algebra and Geometric Figures
C-5 Trigonometric, Exponential and Logarithmic Functions
C-6 Functions and Changes of their Values

The contents of these topics are sketched below.

C-1 Quadratic Functions

1. Function (I)

Linear function $y=ax+b$ and its graph

Quadratic function $y=ax^2$ and its graph

2. Equation

Expansion of $(ax+b)^2$

Factoring of quadratic trinomial ax^2+bx+c

Quadratic equation $ax^2+bx+c=0$

Discriminant of quadratic equation

3. Function (II)

Quadratic function $y=ax^2+bx+c$ and its graph

Equations (linear and quadratic) and graphs of functions

4. Applications of the graph of functions

Inequality $ax+b>0$ ax + b > 0

Inequality $ax^2+bx+c>0$

Maximum, minimum

Notes for discussions

(a) To what extent are traditional topics such as expansion and factoring to be included?

(b) Quadratic equations with imaginary roots are to be excluded.

(c) A vital point in this section is how to germinate the concept of function and its applicability.

C-2 Finite Mathematics

1. Finite sequence

Idea of sequence

Arithmetic sequence $a_{n+1}=a_n+q$

Geometric sequence $a_{n+1}=pa_n$

Sequence $a_{n+1}=pa_n+q$

2. Sum of sequences

Sum of an arithmetic sequence

Sum of a geometric sequence

Sum of squares of natural numbers

3. Elements of combinatorics

Law of product, law of sum

Permutation ${}_nP_r$, circular permutation

Combination ${}_nC_r$ and its properties ${}_nC_r={}_nC_{r-1}+\cdots$

Binomial expansion

4. Probability

Introduction to discrete probability

Expectation

Notes for discussions

(a) Is it necessary to include $a_{n+1}=pa_n+q$?

(b) Is it appropriate to include the binomial expansion?

(c) Use of calculators should be recommended.

C-3 Measurement of Geometric Figures

1. Trigonometric ratios

tan α , sin α , cos α

Basic relations among trigonometric ratios:

Sum of squares, tan α in terms of sin α , cos α , complementary-angle relations

2. Sides and angles of a triangle

Sine theorem for acute triangles

Cosine theorem for acute triangles

Area of triangles $S = bc (\sin A)/2$

3. Figures in space

Length of segments and magnitudes of angles in simple particular figures in space (in order to foster familiarity with spatial figures)

Notes for discussions

(a) Which domain of angles is appropriate in dealing with trigonometric ratios?

$0 < \alpha < 90°$, $0 < \alpha < 180°$, $0 < \alpha < 360°$

(b) To what extent should the synthetic approach to Euclidean geometry be reviewed in connection with geometry in the lower secondary school mathematics?

(c) Main emphasis in this section is placed upon approaches to geometric figures through the work of measuring.

C-4 Algebra and Geometric Figures

1. Equation of a straight line

Coordinates, length, points on a segment, midpoint, centroid

Equation of a straight line $y = m(x - a) + b$

Relations between two straight lines, simultaneous linear equations

2. Equation of a circle

Equation of a circle $(x-a)^2 + (y-b)^2 = r^2$

Tangent to a circle (meaning of double roots)

Circles and straight lines, simultaneous equations (geometric meaning)

3. Locus and graph of conditions

Locus of a moving point, equation and its graph

Domain of a moving point, inequality and its solution set

4. Introduction to vector

Vectors and their operations

Vectors and geometric figures

Vectors in a coordinate plane (components)

Vector equations of a straight line

Vectors in space

Notes for discussions

(a) Careful selection of contents is particularly necessary as these topics are most overloaded. What should be included here and what should be transferred to the option? Need for good problems. How to gather them?

(b) Trigonometric ratios and trigonometric functions should be studied in separate setting.

(c) Main theme of this section is treatment of figures using algebra. How is this most effectively realized?

C-5 Trigonometric, Exponential and Logarithmic Functions

1. Trigonometric function

six x, cos x, tan x for general x

Fundamental relations (with minimum number of formulas)

Graph, periodicity

2. Addition theorem

Proof of the theorem

Applications to geometric problems

Formula for the double angle and its applications

3. Exponential function

Integral power, law of exponents

Fractional exponent, $y = a^x$

Image of the exponential function

Application of exponential functions

4. Logarithmic function

Definition of $\log_a M$ $(a > 1)$

Laws of logarithm and applications

Graph of logarithmic functions

Notion of inverse functions

Notes for discussions

(a) Is it possible to include the addition theorem in the core?

(b) Is it necessary to include logarithmic functions with its base less than 1?

(c) We should not introduce logarithm through the idea of inverse function, but should exemplify the idea of inverse through logarithm. Is this true?

C-6 Functions and Its Change of Values

1. Quadratic and cubic functions

Tangent

Increase and decrease of functions and the sign of differential coefficients

Derivative $(x^n)' = nx^{n-1}$ $(n = 0, 1, 2, ...)$

Graphs of quadratic and cubic functions

2. Fractional function and some others

$$y = a/x, \quad y = a/(x-p) + q, \quad (1/x)' = -1/x^2, \quad (\sqrt{x})' = 1/(2\sqrt{x})$$

Change of values of $y = ax + b/x$, $y = ax$

3. Applications of differentiation

Maximum and minimum

Rate of changes in general

Notes for discussion

(a) What about integration? Should it be a core topic?

(b) Differentiation of $y = x^n$ with a general rational number n would be too difficult to be included here

(c) How about approximate expressions of simple functions such as $1/(1-x) = 1 + x + x^2 + \ldots$ and $\sqrt{1+x} = 1 + (1/2)x + \ldots$

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APPENDIX

List of candidates for topics to be included as option-modules.

1. Equations (side option)
2. Inequalities (side option)
3. Complex Number
4. Mathematical Induction (side option)
5. Geometry in Space (side option)
6. Conic Section
7. Area and Volume
8. Series
9. Differentiation
10. Definite Integral
11. Differential Equations
12. Application of Probability
13. Computer
14. Dealing with Many Variables (side option)
15. Set, Proof, Mapping (side or advanced option)
16. Development of Mathematics (side option)
17. Integer, Residue System, Binary System
18. Numbers and their Properties (remedial option)
19. From Number to Letters (variables) (remedial option)
20. Construction of Geometric Figures

Items without a specified option are for advanced options.

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